FamiliesAndSocieties
Working Paper Series
Changing families and sustainable societies:
Policy contexts and diversity over the life course and across generations

29 (2015)

Mother's Time Allocation, Child Care and Child Cognitive Development

Ylenia Brilli

A project funded by European Union's Seventh Framework
Programme under grant agreement no. 320116

Mother's Time Allocation, Child Care and Child Cognitive Development

Ylenia Brilli[1]

Abstract:
This paper analyzes the effects of maternal employment and non-parental child care on child cognitive development, taking into account the mother's time allocation between leisure and child-care time. I estimate a behavioral model, in which maternal labor supply, non-parental child care, goods expenditure and time allocation decisions are considered to be endogenous choices of the mother. The child cognitive development depends on maternal and non-parental child care and on the goods bought for the child. The model is estimated using US data from the Child Development Supplement and the Time Diary Section of the Panel Study of Income Dynamics. The results show that the productivity of mother's child-care time substantially differs by a mother's level of education. Moreover, the child-care time of college-educated mothers is more productive than non-parental child care. The simulation of maternity leave policies, mandating mothers not to work in the first two years of the child's life, reveals that the impact on the child's test score at age five is either positive or negative, depending on whether the leave is paid or not. The heterogeneous productivity of mothers' time leads to different allocation choices between child care and leisure: college-educated mothers re-allocate a larger fraction of their time out of work to child care than do the lower educated, while the opposite holds for leisure.

Keywords: mother employment, mother time allocation, non-parental child care, child development, structural estimation

Affiliation:
Max Weber Programme, European University Institute.

Acknowledgement: The research leading to these results has received funding from the European Union's Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 320116 for the research project FamiliesAndSocieties.

[1] I am indebted to Daniela Del Boca for her constant encouragement and advice. I am also grateful to Christopher Flinn, Cristian Bartolucci, Matthew Wiswall, Andrea Ichino, Jerome Adda, Marco Francesconi, Chiara Monfardini, Silvia Pasqua, Eva Moerk, James Heckman, Stefano Mosso, Frank Vella,seminar participants at University of Turin, Collegio Carlo Alberto, EUI, University of Lausanne and participants at the XI Brucchi Luchino Labor Economics Workshop (Trento), the XIII MOOD Workshop(EIEF, Rome), the II Summer School on Socio-Economic Inequality (University of Chicago), the 2013 EEA Congress (University of Gothenburg) and the XIII IZA/SOLE Transatlantic Meeting for Labor Economists (Buch am Ammersee) for their useful comments and suggestions. I also thank Matthew Wiswall for sharing his computer codes. This research was initially conducted at New York University and at Collegio Carlo Alberto: the hospitality of both institutions is gratefully acknowledged. I also acknowledge financial support from Collegio Carlo Alberto (project 'Parental and Public Investments and Child Outcomes') and from the European Union Seventh Framework Programme under grant agreement no. 320116 for the research project FamiliesAndSocieties. Any errors should be attributed to the author.

1. Introduction

There has been a long-standing interest in the social sciences literature in learning about the production of child cognitive achievement. Psychologists and economists agree that one of the most valuable inputs for child development is the time the child spends with the mother (Cunha et al. 2006). The increase in the maternal employment rate and the associated rise in the use of non-parental forms of child care have raised concerns about the impact they might have on child development, in particular through the decline in maternal child-care time. In the United States, the participation of mothers in the labor market has increased from around 50 percent in the 1970s to more than 70 percent at the end of the 1990s (U.S. Census Bureau 2000), while, in the same period, the fraction of 3 to 5 year old children enrolled in some forms of non-parental child care programs increased from 7.9 to 51.7 percent for mothers in the labor force (Bianchi 2000). However, recent data from the American Time Use Survey show that, while employed mothers work on average five hours per day, the time spent with their child is only half an hour lower than that of non-employed mothers (U.S. Census Bureau 2013). Moreover, employed mothers are found to spend a substantially lower amount of time in activities, such as socializing, doing sport or watching TV, usually defined as *leisure* (U.S. Census Bureau 2013). This suggests that there might not be a one-to-one corresponding relationship between time spent at work and child-care time, and that mothers not only decide about their labor supply and non-parental child care use, but also about how much of their time out of work should be spent with their child instead of engaging in leisure activities.

This paper analyzes the effects of maternal employment and non-parental child care on children's cognitive development, distinguishing between maternal care and care provided by market services, and taking into account the additional choice between leisure and time with the child.[1] I estimate a behavioral model, in which maternal labor supply and time allocation, as well as non-parental child care and expenditure for the child, are considered to be the endogenous choices of the mother. The child development process depends on the mother's child-care time, goods bought for the child and the amount of time the child spends in non-parental child care. The estimation of such a model makes it possible to deal with the endogeneity and the simultaneity of all the mother's choices, and to identify the contributions of both maternal child-care time and non-parental child care for the cognitive development of the child.

There have been several studies assessing the effects of maternal employment or non-parental child care use on the subsequent cognitive development of children,[2] but only Bernal (2008) evaluates the impact of the two simultaneous choices using a structural approach. Bernal (2008) reports that one year of maternal employment and non-parental child care reduces the child's test scores by 1.8 percent, suggesting a substantial negative effect of both choices. However, the author does not consider the third choice the mother

[1]Non-parental child care includes any type of child-care arrangement provided by people or institutions outside the family, such as child-care centers, babysitters or other types of informal arrangements.

[2]See Ermisch and Francesconi (2005) for a review of studies assessing the effects of maternal employment on children's development, and Bernal and Keane (2011) for a review of studies looking at the impact of non-parental child care services in the U.S.

can make regarding her time allocation between time with the child and leisure, and instead assumes that a mother's time out of work is entirely spent by the mother with the child. Indeed, employed mothers may allocate their time out of work in such a way as to give priority to the time spent with the child (Bianchi 2000; Hoffert and Sandberg 2001). Recent studies have exploited the information on the actual amount of time spent by the mother with the child, also used for this paper, to assess the effects of maternal time inputs on child development, although they do not consider the role played by non-parental child care (Del Boca, Flinn, and Wiswall 2014; Del Boca, Monfardini, and Nicoletti 2012). This paper also refers to a recent body of literature assessing the effects of maternity leave policies on the subsequent development of the new-born child, which has offered mixed results: estimates from these studies range from longer maternity leaves being beneficial (Carneiro, Løken, and Salvanes 2014), to having a null effect (Baker and Milligan 2010; Dustmann and Schönberg 2011; Rossin-Slater 2011).[3]

The contribution of this paper to the literature is threefold. First, I estimate a model incorporating four endogenous choices of mothers' time allocation and investments decisions on the child, namely maternal labor supply, maternal child-care time, non-parental child care use and goods expenditure for the child. The model imposes no restrictions on the relationship between mother's labor supply and mother's child-care time: it allows a direct estimation of the impact of maternal time on a child's development, accounting for the fact that the mother not only chooses how many hours to work and how much time to use non-parental child care, but also how much time to devote to the child instead of engaging in leisure activities. To this purpose, this paper exploits the actual measure of maternal time instead of using a proxy, hence allowing all the mother's choices to be treated as endogenous. Second, this paper represents the first attempt to estimate the elasticity of a child's ability with respect to both maternal time and non-parental child-care time. To the best of my knowledge, there are no studies that simultaneously evaluate the productivity of both mother and non-parental child care, taking into account the selection of mothers into work and child care use. To this end, the paper is also linking in a novel way data on mother's child-care time with information on non-parental child care use. Third, the paper connects the literature looking at the effects of maternal employment on children's cognitive outcomes with recent works evaluating the effects of maternity leave policies. In fact, the model is used to simulate the effects of policies aimed at increasing the amount of time spent by the mother with the child after childbirth, in order to uncover their implications in terms of mother's time allocation, non-parental child care use and expenditure for the child.

In the model presented in this paper, the mother's utility maximization problem is subject to the mother's time and budget constraints, as well as the child's cognitive ability production function. The mother cares about consumption, leisure and the child's

[3]More precisely, Baker and Milligan (2010) and Rossin-Slater (2011) find little evidence that the introduction of maternity leave provisions in Canada and the U.S. has affected children's health and cognitive development. Similarly, Dustmann and Schönberg (2011) do not report any effects of a longer paid leave in Germany on the educational attainments of children, but find negative effects in case of unpaid leaves. In contrast, Carneiro et al. (2014) find positive effects of a longer paid maternity leave implemented in Norway on children's final education.

cognitive ability, while child's ability is specified with a value-added functional form and depends on a mother's child-care time, the goods bought for the child and the amount of time the child spends in non-parental child care. In each period, the mother decides her own labor supply and the investments in the child development process. The empirical specification of the model takes into account that mothers who work and use non-parental child care are systematically different from those who do not, and also that mothers' unobserved ability can be genetically transmitted to their children. Moreover, the model allows mothers to differentially allocate their time between labor, time with the child and leisure, according to their preferences, their productivity in the labor market and their productivity in the child development production process.

The model is estimated using U.S. data from the Panel Study of Income Dynamics (PSID), linked to data from the Child Development Supplement (CDS) and the Time Diary (TD) Section. The CDS provides retrospective information on all child-care arrangements used since birth, while the Time Diary (TD) component provides unique data on the amount of time the child spends with the mother. The main PSID surveys give detailed information on the mother's work history and household income during the child's life cycle. The parameters of the model are retrieved using a Method of Simulated Moments estimator, which minimizes the distance between several data statistics and their model counterparts.

The results show a substantial heterogeneity in the productivity of maternal time with the child by the mother's level of education: maternal child-care time of college-educated mothers is found to be more productive than that of low educated mothers, though this difference fades out as the child grows up. Moreover, the productivity of maternal time with the child for lower educated mothers is not statistically different from the productivity of non-parental child care. This implies that mothers with college education have higher gains from substituting their time to non-parental child care, because the productivity of the alternative form of care is much lower than theirs. This result recalls not only the findings from previous studies on human capital accumulation, suggesting the importance of investments received during early childhood (Heckman 2008), but also recent evidence on the effectiveness of educational activities during the first years of the child, which are performed to a larger extent by highly educated mothers.[4]

The estimated model is used to simulate the effects of policies mandating the mothers not to work in the first two years of the child life, characterized by different types of payment scheme: an unpaid leave is found to have detrimental effects on children's test scores at age five, while a paid leave has positive effects. The unpaid leave has negative effects because the mother's time out of work does not entirely match with the actual time spent by the mother with the child; furthermore, the absence of mother's labor income, through

[4]For instance, Raikes et al. (2006) and Hale et al. (2011) report a positive effect of maternal bookreading and language-based bed-time routine (reading a story, telling a story, praying, talking, singing, etc) in the first three years of the child's life on the cognitive and language development of the child measured between age three and five. Recently, Kalb and Van Ours (2014) and Price (2010) have given a causal interpretation to the relationship between the time the parents spend reading to the child and the child subsequent development. From these studies, it also emerges that the probability of mothers reading to their children or performing language-based activities is higher for high educated mothers.

which the mother can invest in her child's development either buying goods or choosing non-parental child care, also plays a role. The heterogeneous productivity of mothers in the child development process and in the labor market yields different time allocation between child care and leisure, as well as different responses in terms of expenditure and non-parental child care use: in a paid leave policy scheme, in which the payment is proportional to the mother's wage, college-educated mothers increase their expenditure for the child and the non-parental child care use by a greater amount than the less educated mothers. Finally, the policies are found to have no effects in the long run, both because of the diminishing productivity of maternal and non-parental child care and because the difference in the productivity of mother's time by the mother's level of education progressively fades out. The results of the policy simulations shed light on the effects of leave policies on mother's time allocation and investment decisions, and are able to explain why recent studies looking at the effects of maternity leave policies do not find long term effect on the child cognitive development.

The rest of the paper is organized as follows. Section 2 presents the related literature and key stylized facts in non-parental child care use and maternal time allocation. Section 3 presents the model that is estimated, while Section 4 describes the data. Section 5 presents the empirical method used for the identification of parameters, while Section 6 presents the results and discusses the goodness of fit of the model . Section 7 presents the results from the policy simulations and Section 8 concludes.

2. Background

The increase in the female employment rate that has characterized all developed countries has raised concerns about the impact that maternal employment and non-parental child care may have on child development. Since the work of Becker and Tomes (1986), who first provided a framework for modeling the implications of household decisions for children's subsequent utility and earnings, there has been a growing literature on the impact of parental investments on children's human capital and development. Studies on maternal employment and non-parental child care present mixed findings. Several reduced-form studies find negative effects of maternal employment (Baydar and Brooks-Gunn 1991; Belsky and Eggebeen 1991; Desai, Chase-Lansdale, and Michael 1989; Ruhm 2004), while others find null effects (Chase-Lansdale et al. 2003; James-Burdumy 2005; Parcel and Menaghan 1994). Bernal and Keane (2011) analyze the effects of attending any type of non-parental child-care arrangement in the U.S. and find that one year of child care use decreases children's cognitive outcomes by 2.13 percent. In contrast, Magnuson, Ruhm, and Waldfogel (2007) find positive effects of having attended pre-kindergarten on academic achievement at kindergarten and primary school. Loeb et al. (2007) find that staying in center-based child care for more than 15 hours per week increases reading and Math score by almost 8 and 7 percent of a standard deviation.

The identification of the effects of both maternal employment and non-parental child care on child development is hampered by several sources of endogeneity, induced by the correlation of mothers' choices with mothers' and children's unobserved characteristics, as well as by the simultaneity of these choices. While the empirical reduced-form strategies usually adopted in this literature (such as Instrumental Variables or Mother fixed effects estimators) can solve this issue for one endogenous choice, these techniques hardly solve the issue of the simultaneity between the employment and non-parental child care decisions.

Structural estimation makes it possible to account for the sources of endogeneity that may arise in this context, modeling the mother's decision-making process for different choices. In this framework, each input is optimally chosen by the mother who maximizes her own utility function, with the child's ability as an argument, and the child's ability production function is one of the constraints to this maximization problem. There are few studies using structural estimation in the child development literature. The model presented in this paper builds on Del Boca, Flinn, and Wiswall (2014), who model household choices and investments in child ability from childbirth up to adolescence. They find that the productivity of a mother's time investments declines over a child's age, and that a father's time becomes more productive as the child reaches adolescence. Differently from Del Boca et al. (2014), this paper does not model both parents' labor supply and time allocation decisions, focusing instead on mothers' behavior and on the additional choice of using non-parental forms of care; in other words, instead of considering fathers' time as a substitute for mothers' time with the child, the present study analyzes the role of non-parental child-care time as a substitute for maternal child care.

Mroz, Liu, and Van der Klaauw (2010) specify and estimate a behavioral model of household migration and maternal employment decisions in order to assess the effect of these choices on a child's cognitive ability. They find that part-time employment of the mother reduces the child's score by 3 percent of a standard deviation while the mother's full-time status reduces the score by 5 percent of a standard deviation. Recently, Ermish and Francesconi (2013) have evaluated the effects of maternal employment on a child's schooling, estimating the parameters of a conditional demand function for the child's education; they find that one year more of a mother's full time employment reduces the probability that the child reaches higher education by 11 percentage points.

Bernal (2008) is the only study that evaluates the impact of both maternal employment and non-parental child care attendance on subsequent child outcomes using a structural approach. She finds that one year in external child care reduces the child's cognitive ability by 0.8 percent; however, the impact of maternal employment and non-parental child care is more detrimental, since, together, they decrease a child's test score by 1.8 percent.

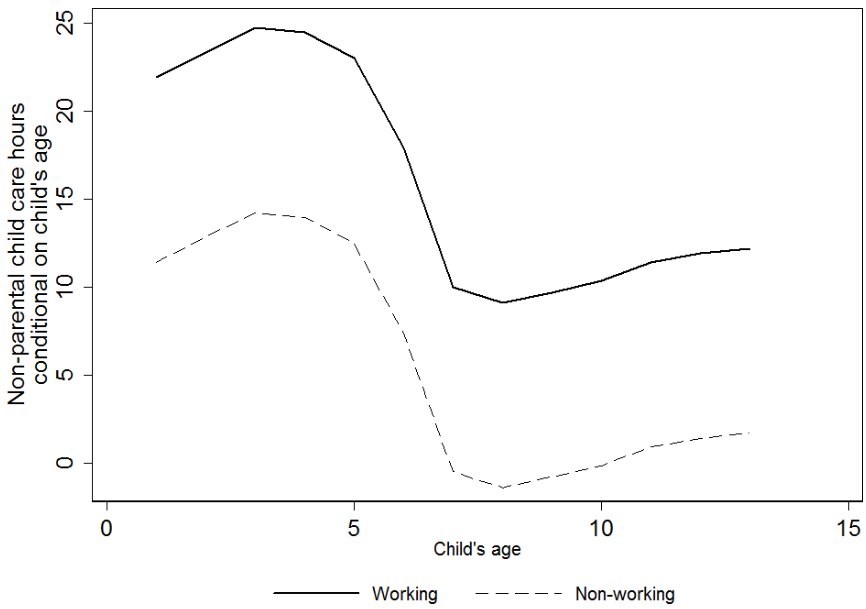
Exploiting the actual measure of maternal time with the child and the detailed information on non-parental child care use, provided by the combined CDS-TD data of the PSID, this paper assesses the effects on children's cognitive development of maternal employment and non-parental child care use, accounting for the additional choice the mother makes between child-care time and leisure and for the potential positive effects of mother's work induced by the higher expenditure for the child. Notice that the assumption concerning mothers' time allocation used in the previous studies may have implications for the effect that is actually estimated. In fact, arguing that maternal time with the child can be proxied by the amount of time the mother spends out of work rules out the possibility that mothers choose how to allocate their time between leisure and child-care time. Indeed, the actual investment made by the mother on the child through her contact time may differ according to how the mother allocates her time between leisure and child-care time.

Even though data on mothers' and children's time use have become available only very recently, there have been some studies suggesting that mothers do not differ only in terms of participation decisions but also in terms of the allocation of leisure and child-care time. For instance, Leibowitz (1974, 1977) points out that more skilled mothers may also have a higher propensity to stay with their child, even if working. Recent studies on mothers' time use confirm this point, since they do not find significant differences across employment status in the amount of time mothers spend with their child (Bianchi 2000; Hoffert and Sandberg 2001). Two main reasons may explain the absence of significant differences in maternal time with the child between working and non-working mothers. First, during recent years, non-working mothers have also started using non-parental child care, so that children of non-working mothers may not be always available for maternal investments while attending external child care. For instance, Bianchi (2000) shows that from the end of the 1960s to the end of the 1990s, the fraction of 3 to 5 year old children enrolled in some forms of pre-primary educational programs increased from 4.8 to 44 percent for mothers not in the labor force. Second, working and non-working mothers may allocate their time out of work differently, so that the actual time that they spend with the child does not correspond to the time they spend out of work. Guryan, Hurst, and Kearney (2008), exploiting ATUS data for 2003-2006, find that there is a striking positive education and income gradient in child care, while the gradient for leisure is negative: this means that more educated and wealthier mothers spend more time with their child even if working, preferring to renounce some leisure time. According to data from the American Time Use Survey (ATUS) 2005-2009, the amount of time spent by mothers reading and playing with the child does not vary substantially across employment status: while employed mothers work, on average, five hours per day, they spend with their child only 30 minutes less than their non-employed counterpart; in contrast, employed mothers spend, on average, 2.5 hours per day in activities like socializing, doing sports or watching TV, against the 4 hours per day spent by not-employed mothers (U.S. Census Bureau 2013).

Descriptive evidence from the data used in this paper supports the existence of these patterns. Figure 1 shows that non-working mothers also use a positive amount of non-parental child care for their child. This may happen if, for instance, they value the educational role of the service and choose it as an investment in their child's human capital. However, since the difference in average child-care time between working and non-working mothers is equal to 10.50 hours per week, the graph also confirms that non-parental child care is needed for its custodial purposes anytime the mother is working.

Figure 2 plots the fitted values of two regressions where the dependent variables are, respectively, maternal child-care time and leisure time. The graph on the left (i.e., maternal child-care time) confirms that employed mothers allocate their time out of work in order to spend a positive amount of time with their child. Conversely, non-working mothers do not spend all their time with the child, but only around 30 hours per week when the child is very young and around 25 when the child grows up. The graph on the right shows the fitted values of a regression on a child's age fixed effects where the dependent

FIGURE 1
Non-parental child-care time by mothers' employment status.



NOTE. The vertical axis represents the fitted values of the following regression:

$$i_{it} = \eta_0 + \sum_{t=1}^{T} \eta_{1t} t_{it} + \eta_2 d_{it} + \epsilon_{it}$$

where i_{it} represents (weekly) hours of non-parental child care in each year t, t_{it} are child's age fixed effects (with $t = 1, \ldots, 12$), d_{it} is a dummy variable equal to 1 if the mother of child i works in period t. $\eta_2 = 10.50$ represents the difference in average child care use (conditional on child's age) between working and non-working mothers. Source: own elaboration from PSID-CDS data ($N = 3381$).

variable is leisure time, computed as the difference between the total time endowment and the sum between working time and time with the child. Employed mothers spend a lower amount of time out of work in leisure, while the corresponding level for non-working mothers is considerably higher. Notice that while the difference in maternal time with the child between working and non-working mothers is equal to 8 hours per week, the difference in leisure is equal to 27 hours per week. These patterns suggest that working and non-working mothers allocate their time out of work differently and that the choice of devoting time to the child instead of having leisure should be considered endogenous as those of labor supply and non-parental child care use.

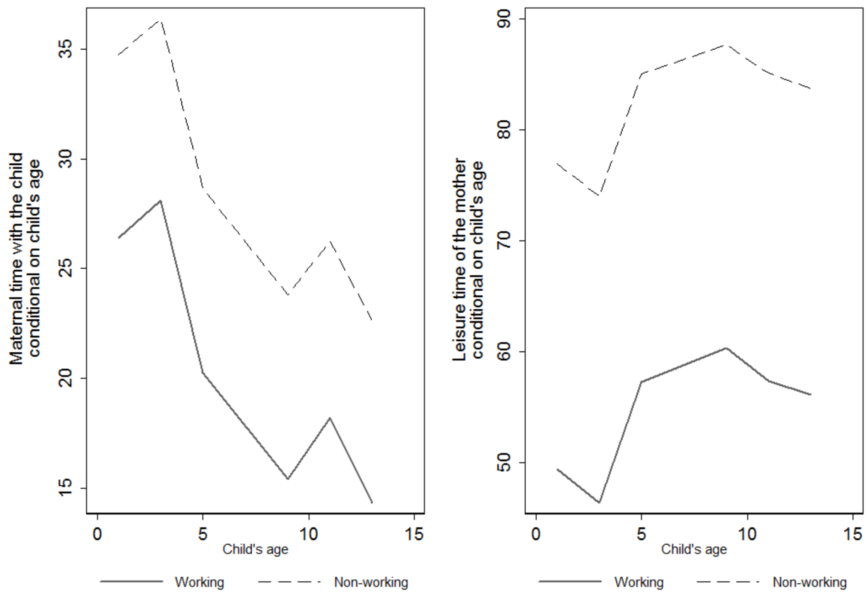
3. THE MODEL

This section describes the model that is estimated: paragraph 3.1 presents the basic structure, while paragraph 3.2 derives the demand functions for all the choice variables; paragraph 3.3 describes the empirical specification.

3.1. ***Basic structure.*** The model follows a standard framework from Becker and Tomes (1986), where household preferences are described by a unitary utility function, with child's ability as an argument, and subject to a production function for child's ability and budget

FIGURE 2

Maternal child-care time and leisure by mothers' employment status.



NOTE. The vertical axis in the graph on the left represents the fitted values of the following regression:

$$\tau_{it} = \eta_0 + \sum_{t=1}^{T} \eta_{1t} t_{it} + \eta_2 d_{it} + \epsilon_{it}$$

while the vertical axis in the graph on the right represents the fitted values of the following regression:

$$l_{it} = \beta_0 + \sum_{t=1}^{T} \beta_{1t} t_{it} + \beta_2 d_{it} + \varepsilon_{it}$$

τ_{it} stems for (weekly) maternal time with the child and l_{it} represents leisure time, computed as $l = TT - \tau - h$, where $TT = 112$ is the total time endowment and h represents weekly hours of work. t_{it} are child's age fixed effects (with $t = 1, \ldots, 12$) and d_{it} is a dummy variable equal to 1 if the mother of child i works in period t. $\eta_2 = -8.32$ represents the difference in average maternal time (conditional on child's age) between working and non-working mothers. $\beta_2 = -27.49$ represents the difference in average leisure time (conditional on child's age) between working and non-working mothers. Source: own elaboration from PSID-CDS data ($N = 380$).

and time constraints. The functional form assumptions are based on the theoretical model developed in Del Boca et al. (2014).

The model is dynamic and evolves in discrete time. In each period, the mother decides her own labor supply and time allocation, as well as the amount of non-parental child care to use and the level of expenditure in goods for the child. The choice variables are then: (i) h_t, representing hours of work; (ii) τ_t, the time the mother spends with the child; (iii) i_t, hours of non-parental child care and (iv) e_t, the expenditure in goods bought for the child. The timing is defined as follows: $t = 0$ represents the birth of the child and the mother makes all the decisions at each child's age t until the child reaches T years of age.[5]

The mother is the unique decision maker in the household concerning investment decisions on the child. This assumption implies that the father's time allocation is exogenous

[5] $t = 1$ indicates the first 12 months of the child's life, $t = 2$ refers to the next 12 months of the child's life, and so on. $t = T = 13$ represents the terminal period of the model. It may be interpreted as the final period of middle childhood before the child enters adolescence.

with respect to mother's choices and to the child development process.[6] The model applies to intact households, where both the mother and the father are present, and only households with one child are considered.[7]

The Mother's Utility Function

The mother's utility in each period is a function of her own leisure time (l_t), i.e, the time the mother spends alone without working, household consumption (c_t), including father's and child's consumption, and the child's cognitive ability (A_t). I assume a Cobb-Douglas form for preferences and I restrict the preferences parameters to be stable over time:

$$u(l_t, c_t, A_t) = \alpha_1 ln l_t + \alpha_2 ln c_t + \alpha_3 ln A_t \quad (1)$$

where $\sum_{j=1}^{3} \alpha_j = 1$ and $\alpha_j > 0$, $j = 1, 2, 3$.

The mother maximizes her utility subject to the budget and the time constraints. The budget constraint takes into account household consumption and expenditure for the child, as well as the total income available in the family (from both parents' labor supply and non labor income); it is given by:

$$c_t = w_t h_t + I_t - p i_t - e \quad (2)$$

where w_t is mother's hourly wage; I_t represents household earnings (including father's labor income and household non labor income); i_t represents the number of hours that the mother uses non-parental child care and p is the hourly price of child care; e represents the expenditure in goods bought for the child, whose price has been normalized to 1. The variable i_t includes any kind of non-parental child-care arrangements and all contributions to child development due to the alternative care providers' time.[8] Hence, it is assuming that the mother's decision-making process for the two types of care is similar. The same homogeneity is then reflected in the price of non-parental child care. The model predicts a strictly positive price of the service, regardless of its nature. This implies that services with a potentially zero price in the market are also characterized by a shadow price, representing, for instance, the limited availability of informal care or the value of the

[6]The model allows the father to affect child development in two ways: first, the child's ability endowment depends also on the father's education; second, the father's labor income contributes to household earnings, which influence mother's choices concerning work, non-parental child care and time with the child and also affect the level of expenditure for the child. In the sample of intact households that I use for the estimation of the model, all fathers work and the average working time does not change across mother's participation decisions; moreover, a father's time with the child does not vary across a mother's employment (see Figure D.1 in Appendix D.1). Appendix D.1 shows the results of a sensitivity analysis where the time investments received by the child include both mother's and father's time.

[7]In the data used to estimate the model, the sample of intact households represents 52.7 percent of the overall sample, while 36.2 percent of families in the sample has only one child. These figures roughly correspond to the official statistics: according to the US Census Bureau data (2012), 68.1 percent of children under 18 live with both parents and 47.8 percent of married women live with their spouse; finally 21.6 percent of married women have only one child. However, the sample selection may have implications for the estimated parameters; this issue will be further discussed in Section 4.

[8]More precisely, non-parental child care includes any type of arrangement used by the mother for the child, excluding parents' (mother's and father's) child care; this can include formal child-care centers, nannies and babysitters, as well as grandparents. When the child reaches school age, this variable indicates any forms of preschool or after school programs, or any other informal arrangement used by the mother in addition to formal schooling.

unpaid care provider's time in alternative activities (Blau and Currie 2006; Ribar 1992).[9] Finally, the mother does not make saving decisions, hence household income defined by I_t can be considered exogenous with respect to all the mother's choices.

The time constraint is defined as:

$$TT = l_t + h_t + \tau_t \tag{3}$$

where TT is the mother's total time endowment.[10] Notice that, in each period, the mother can choose to spend her leisure time alone (l_t) or to devote some time to the child (τ_t): hence, the model allows the mother to further choose between leisure and time with the child when she is not at work.

The Child's Cognitive Ability Production Function

The child's cognitive ability production function (hereafter CAPF) is defined using a value-added specification and taking a Cobb-Douglas form:

$$lnA_{t+1} = \delta_{1t} ln\tau_t + \delta_{2t} lni_t + \delta_{3t} lne_t + \delta_{4t} lnA_t \tag{4}$$

where A_{t+1} is the outcome for a child at time $t+1$, τ_t, i_t and e_t are the inputs decided by the mother in each period t, where τ represents the amount of time the mother spends with the child, i the amount of time in non-parental child care and e the level of expenditure in goods for the child; A_t is the level of child ability at period t. Since current ability influences the child's future ability, equation (4) shows that inputs operate with a lag. Moreover, the structure of the CAPF implies that when deciding the inputs on child development, the mother knows the productivity of each of them and the level of a child's ability in the previous period.

Despite posing some limitations on the substitution pattern across inputs because of the assumed functional form, the model allows the parameters in (4) to vary across the age of the child in order to capture the fact that marginal productivity of inputs varies over the stages of child development (Cunha, Heckman, and Schennach 2010; Heckman 2007). Moreover, as shown in paragraph 3.3, a mother's time productivity is also allowed to vary across a mother's educational level.

A mother's work is not explicitly included in the CAPF, because it may not have a direct impact on child development *per se*. A mother's employment may indirectly affect child development through a change in her time allocation, together with the use of non-parental child care. Furthermore, allowing also the expenditure in goods for the child to be an endogenous variable, the model takes into account the potentially beneficial effect of a mother's work induced by the increased available income in the household. This specification makes it possible to test whether, in each period, maternal time is more

[9] In the empirical analysis, the hourly price of non-parental child care p is estimated, because the actual distribution of that measure in the data has a large mass toward zero, also for children actually using the service. This may be due to the use of informal child care, that can have a zero market price. Using the direct measure available in the data yields an infinite demand for external child care for those using an arrangement with a zero price, regardless of a mother's labor income and household earnings.

[10] $TT = 112$ hours per week. All choice variables are defined on a weekly basis.

productive than non-parental child-care time. If this is the case, then, for any period and for an equal amount of maternal time and child-care time used, $\delta_{1t} \geq \delta_{2t}$.[11]

Maximization Problem

In each period, the mother maximizes her expected life time utility, optimally choosing her labor supply, the child care and expenditure inputs and the number of hours to devote to the child. In this decision-making process the mother takes into account the level of ability reached by the child in each period, the wage offer that she receives from the market and the level of income in the household. The child's cognitive ability represents an endogenous state variable, while the wage offer the mother receives in each period and household income are exogenous with respect to the maximization problem but differ for each mother in each period. The initial condition of the problem is given by the value of the state variables in the first period.[12]

The value function for the mother at period t is given by:

$$V_t(S_t) = max_{h_t,\tau_t,i_t,e_t} \;\; u(l_t, c_t, A_t) + \beta E_t V_{t+1}(S_{t+1}) \tag{5}$$

$$s.t. \;\; c_t = w_t h_t + I_t - p i_t - e_t$$

$$TT = l_t + h_t + \tau_t$$

$$lnA_{t+1} = \delta_{1t} ln\tau_t + \delta_{2t} ln i_t + \delta_{3t} ln e_t + \delta_{4t} lnA_t$$

where $\beta \in [0, 1]$ and $S_t = \{A_t, w_t, I_t\}$ represents the vector of state variables. The timing of the model implies that after childbirth and during the first 12 months of a child's life the mother observes the initial level of her child's ability and the level of income in the household and receives a wage offer; then she makes her decisions. Similarly, in the following periods, the mother chooses h_t, i_t, e_t and τ_t after having observed the corresponding level of A_t and I_t and after having received the wage offer from the labor market.

It should be noticed that the maximization problem of the mother can be solved analytically only if the wage offer is exogenous with respect to the mother's past and current labor supply choices. This implies that the offer the mother receives in period t is not affected by her working decisions in $(t - 1)$ and that it does not reflect any depreciation in the mother's productivity as a result of her absence from the labor market after childbirth. The exogeneity of wage is necessary to estimate the model with continuous choice variables and closed-form solutions, which is needed to allow for four choices and to take into account the additional choice between leisure and time with the child. However, this assumption may have implications on the estimated parameters. In fact, since the definition of the wage process does not take into account the potentially negative effect on wages of leaving the labor market after childbirth, it is very likely to overestimate the

[11] For any period t, the marginal productivity of maternal time is given by $MP_{\tau_t} = \frac{\delta_{1t}}{\tau_t}$, while the marginal productivity of non-parental child care is $MP_{i_t} = \frac{\delta_{2t}}{i_t}$. For $\tau_t = i_t$, $MP_{\tau_t} \geq MP_{i_t}$ if $\delta_{1t} \geq \delta_{2t}$; viceversa, $MP_{\tau_t} \leq MP_{i_t}$ if $\delta_{1t} \leq \delta_{2t}$.

[12] The structure of the initial condition for child's ability and the draws from which the initial values of w_t and I_t are taken will be defined in paragraph 3.3.

proportion of mothers working and their labor supply on the extensive margin; this may also lead to an overestimation of the amount of non-parental child care used.

3.2. ***Terminal period value function and solutions of the model.*** The mother makes her decisions (that are relevant for the child development process described by equation (4)) in the first T years of the child's life. After period T, both the mother's optimization problem and the child's ability production function change: the mother may continue to optimally choose labor supply and consumption, but she will no longer consider maternal and non-parental child care choices.

The terminal level of a child's cognitive ability is A_{T+1}, i.e., the level of ability reached in $T+1$, that will not be affected by the mother's subsequent decisions. This level of ability may be interpreted as the starting point for the child's future development during adolescence, from $T+1$ on.

The period $T+1$ maximization problem for an infinitely-lived household may be written as:

$$V_{T+1} = \tilde{V}_{T+1} + \sum_{\kappa=0}^{+\infty} \beta^{\kappa} \alpha_3 lnA_{T+1} \tag{6}$$

where

$$\tilde{V}_{T+1} = max_{h_{T+1}} \; \alpha_1 lnl_{T+1} + \alpha_2 lnc_{T+1} + \beta E_{T+1} \tilde{V}_{T+2}(l_{T+2}, c_{T+2})$$

and $\sum_{\kappa=0}^{+\infty} \beta^{\kappa} = \rho$ represents the value given by the mother to the child's ability in the last developmental period.[13] Equation (6) represents the terminal period value function and implies that the mother's maximization problem after period T becomes stationary and does not depend on the choices made by the mother in the previous periods.

The model is solved by backward induction and yields closed-form solutions for all the choice variables. The solution of the model involves the computation of the value function starting from the terminal period and the corresponding optimal solutions in each period. Following a two-stage process, I first derive the optimal solutions for non-parental child care (i_t), expenditure (e_t) and maternal time (τ_t), conditional on h_t, and then compute the solutions for the mother's labor supply h_t. Analytical derivations of the results are in Appendix A.

The demands for maternal child-care time, non-parental child care and expenditure, conditional on the mother's labor supply, for any period t, are given by:

$$\tau_t^c = \frac{\beta \delta_{1t} D_{t+1}}{(\alpha_1 + \beta \delta_{1t} D_{t+1})} (TT - h_t) \tag{7}$$

$$i_t^c = \frac{\beta \delta_{2t} D_{t+1}}{p(\alpha_2 + \beta \delta_{2t} D_{t+1} + \beta \delta_{3t} D_{t+1})} (w_t h_t + I_t) \tag{8}$$

$$e_t^c = \frac{\beta \delta_{3t} D_{t+1}}{\alpha_2 + \beta \delta_{2t} D_{t+1} + \beta \delta_{3t} D_{t+1}} (w_t h_t + I_t) \tag{9}$$

where $D_{t+1} = \frac{\partial V_{t+1}}{\partial lnA_{t+1}}$ represents the marginal utility the mother gets from the child's future cognitive ability, in each period. The sequence of marginal utilities from period

[13] In the estimation, the discount factor is set at $\beta = 0.95$. In order to increase the flexibility of the model and to allow the discount factor of the mother to differ in the last period of investments with respect to the previous ones, the parameter ρ is estimated.

$T+1$ to period 1 is given by:[14]

$$
\begin{aligned}
D_{T+1} &= \rho\alpha_3 \\
D_T &= \alpha_3 + \beta\delta_{4T} D_{T+1} \\
D_{T-1} &= \alpha_3 + \beta\delta_{4T-1} D_T \\
&\vdots \\
D_t &= \alpha_3 + \beta\delta_{4t} D_{t+1} \\
&\vdots \\
D_2 &= \alpha_3 + \beta\delta_{42} D_3 \\
D_1 &= \alpha_3 + \beta\delta_{41} D_2
\end{aligned} \tag{10}
$$

An implication of the Cobb-Douglas specification used in the mother's utility function and in the child's cognitive ability production function is that all inputs should be strictly positive.[15] However, I do allow for the possibility of corner solutions for the mother's labor supply decisions.

The mother's latent labor supply, conditional on τ_t^c, i_t^c and e_t^c, is given by:

$$
h_t^c = \frac{\alpha_2(TT - \tau_t^c)}{\alpha_1 + \alpha_2} - \frac{\alpha_1(I_t - p i_t^c - e_t^c)}{w_t(\alpha_1 + \alpha_2)} \tag{11}
$$

Substituting (7), (8) and (9) in equation (11), the latent labor supply becomes:

$$
h_t^* = \frac{TT(\alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})}{(\alpha_1 + \beta\delta_{1t}D_{t+1} + \alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})} - \frac{I_t(\alpha_1 + \beta\delta_{1t}D_{t+1})}{w_t(\alpha_1 + \beta\delta_{1t}D_{t+1} + \alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})} \tag{12}
$$

The actual labor supply in each period is determined according to the following rule:

$$
h_t = \begin{cases} h_t^* & \text{if} \quad h_t^* > 0 \\ 0 & \text{if} \quad h_t^* \leq 0 \end{cases}
$$

According to equation (12), the mother's latent labor supply is negative or zero only if household income is strictly positive and sufficiently high. Substituting (12) into (7), (8) and (9) yields the unconditional demands for time with the child, non-parental child care and expenditure.

Notice that a mother's decision to work also depends on the productivity of the alternative forms of care, since if it increases, the mother may be more willing to substitute her time with the external child care provider's time. Equation (8) shows that demand for child care can be driven by necessity of custodial care, i.e., if the mother is working and needs someone to look after the child, or by valuing the educational role of the service. In fact, non-working mothers (for which $h_t = 0$) can demand of non-parental child care if they value the child's ability and they think child care can represent an input for the child's development, as long as the household income is strictly positive and sufficiently high.

---

[14]Notice that the marginal utility in $T+1$ is discounted (through ρ) for all the subsequent periods in which the child's ability does not depend on the mother's investment decisions.

[15]This means that the model always predicts a positive amount of non-parental child care, regardless of a mother's working status or household income.

3.3. ***Empirical specification.*** Unobserved and observed heterogeneity enters any stage of the decision-making process of the mother described in the previous paragraphs. Consider first the mother's utility function, where the parameters, because of the functional form assumptions, should be positive and sum to one. In order to respect these requirements without posing additional constraints on the estimation algorithm, I use a suitable transformation of the original parameters.[16] More precisely, I allow the coefficients in the mother's utility function to vary across the population according to unobserved taste shifters, representing the utility from consumption (γ_2) and the utility from child's ability (γ_3). Thus, the parameters representing the mother's preference for leisure (α_1), consumption (α_2) and child's ability (α_3) are defined as:

$$\alpha_1 = \frac{1}{1 + exp(\gamma_{2k}) + exp(\gamma_{3k})} \tag{13}$$

$$\alpha_2 = \frac{exp(\gamma_{2k})}{1 + exp(\gamma_{2k}) + exp(\gamma_{3k})} \tag{14}$$

$$\alpha_3 = \frac{exp(\gamma_{3k})}{1 + exp(\gamma_{2k}) + exp(\gamma_{3k})} \tag{15}$$

where γ_2 and γ_3 follow a discrete distribution with two points of support ($k = h, l$). The probability that each parameter takes each value ($\pi_{\gamma_{ik}}$, where $i = 2, 3$ and $k = h, l$) should be estimated.

In each period, the mother receives a wage offer and decides whether to enter into the labor market comparing the value of this offer with her reservation wage. The offer the mother receives is described by the following wage equation:

$$ln(w_t) = \mu_t + \epsilon_t \tag{16}$$

where

$$\epsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2_\epsilon)$$

is assumed to be uncorrelated over time and represents a transitory shock on wage. The term μ_t is the mean of the log wage draws of the mother at time t and it is defined as follows:

$$\mu_t = \mu_m + \mu_1 MotherEdu + \mu_2 MotherAge_t + \mu_3 MotherAge_t^2 + \mu_4 MotherRace \tag{17}$$

where μ_m represents the mother's unobserved skills, whose distribution will be specified below. $MotherEdu$ is a continuous variable indicating the mother's years of education, while $MotherRace$ is a dummy variable equal to one if the mother is white.

As for the wage process, the income process is also exogenous with respect to the mother's input decisions in each period. The household income is assumed to have a lognormal distribution and to depend on the fathers' observable characteristics and a shock:

$$ln(I_t) = \mu_{inc0} + \mu_{inc1} FatherEdu + \mu_{inc2} FatherAge_t + \mu_{inc3} FatherAge_t^2 + \mu_{inc4} FatherRace + \iota_t \tag{18}$$

[16]See Mroz et al. (2010) for similar applications.

where $\iota_t \overset{\text{iid}}{\sim} N(0, \sigma^2_{inc})$.

Concerning the child's cognitive ability production function, as stated in Section 3.1, the parameters can vary across a child's age. Moreover, the productivity parameter for maternal time with the child is allowed to vary by the mother's level of education. Thus, the parameters in the CAPF are defined as follows:

$$\delta_{1t} = exp(\xi_{1edu} MomCollege + \xi_{1\tau} t) \tag{19}$$

$$\delta_{2t} = exp(\xi_{2i} t) \tag{20}$$

$$\delta_{3t} = exp(\xi_{3e} t) \tag{21}$$

$$\delta_{4t} = exp(\xi_{4A} t) \tag{22}$$

where $MomCollege$ is a dummy variable equal to one if the mother has completed high school and obtained some college education.[17]

In order to estimate the model and to take into account the dynamic optimization problem faced by the mother, I need to know the starting level of ability, i.e., the child's cognitive ability the mother observes in the first period before making her investments decisions. The initial ability endowment is assumed to be a function of children's unobserved skills, parents' education, child's birth weight and gender. Specifically:

$$A_1 = exp(\psi_{ck} + \eta_1 MotherEdu + \eta_2 FatherEdu + \eta_3 BirthWeight + \eta_4 Male) \tag{23}$$

where $MotherEdu$ and $FatherEdu$ are variables indicating parents' years of education, $Birthweight$ is a dummy variable indicating if a child has a low birth weight and $Male$ is a dummy variable indicating whether the child is a male. ψ_{ck} represents child's unobservable skills.

The specification of the model allows the mother's and the child's unobservable skills to be correlated and to follow a bivariate discrete distribution (Heckman and Singer 1984). More precisely, I assume that both the mother's and the child's unobservable skills distribution have two points of support. The mother's skills are distributed as $f(\mu_m) = P_{mk}$, with $P_{mk} >= 0$ and $\sum_k P_{mk} = 1$; similarly, the child's skills are distributed as $f(\psi_c) = P_{ck}$, with $P_{ck} >= 0$ and $\sum_k P_{ck} = 1$, where $k = h, l$. Assuming two types of mothers and children, and given the bivariate distribution of the mother's and the child's skills, this specification determines four types of children, characterized by a level of ability endowment of the child and a level of ability inherited from the mother. The probability that a child belongs to each type should be estimated, as well as the values taken by each skill level.

Recalling the value-added specification of the CAPF, defined in (4), the estimation provides consistent estimates of the productivity parameters for each input if the following conditions hold: (i) A_t is a sufficient statistic for the inputs history received by the child in the previous periods; (ii) the child's initial endowment A_1 (that the mother observes but

[17] Allowing the parameters to vary across a child's age partially compensates for the lack of substitutability implied by the Cobb-Douglas functional form used to define the CAPF. Moreover, it allows me to capture whether the inputs included in the CAPF become less (more) productive as the child ages and as he receives other inputs, such as schooling. In fact, when the child reaches primary school age, other school inputs can contribute to his own cognitive development and family investments may have a lower influence.

the researcher does not) is only reflected in the level of ability in the subsequent period and does not affect a child's ability in the future periods (Todd and Wolpin 2003).

Finally, it should be described how the child's true cognitive ability is related to the measure of that given by the test scores. Following the approach based on classical test theory (Novick 1966), I define the probability that the child answers correctly to each item as a function of the child's true ability:

$$\pi_{score} = \frac{exp(A_t + v_t)}{1 + exp(A_t + v_t)} \tag{24}$$

where $v_t \overset{\text{iid}}{\sim} N(0, \sigma_v^2)$ represents measurement error capturing the fact that test scores depict true child's ability with a noise. The test score measure is then defined as follows:

$$S_t = \pi_{score} * J_t \tag{25}$$

where J_t is the maximum number of items answered correctly at each child's age.[18]

Summing up, the empirical specification of the model allows the mother's preference parameters to depend on her unobserved tastes, while a mother with higher skills receive, on average, higher wage offer, is more likely to work and to use more non-parental child care. A mother's time productivity differs across educational level, while the mother's and the child's unobserved skills are correlated and affect the initial level of child's ability at birth and the wage offer of the mother.

4. Data

The model is estimated using data from the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS) and Time Diary (TD) component. The PSID is a longitudinal study that began in 1968 with a nationally representative sample of over 18,000 individuals living in 5,000 families in the United States. Starting from 1968, information about each family member was collected, but much greater detail is obtained about the head and the spouse. From 1997, the Child Development Supplement (CDS) has gathered information on children aged 0-12 in PSID families through extensive interviews with their primary caregiver. The CDS has been replicated in 2002 and 2007 for children under 18 years of age.

For this analysis, I exploit the child cognitive ability measures and non-parental child care data provided in the Primary Caregiver Interview of the CDS, together with the time use details given in the Time Diary (TD) component of the CDS. To the best of my knowledge, this is the first study linking all the components of the PSID surveys introduced in 1997 and exploiting the rich information on non-parental child care use provided in the CDS. The main PSID surveys are exploited to recover information on a mother's work and household income.

[18]The score measure used in the empirical analysis is the Letter Word test. To define the thresholds J_t I use the overall PSID-CDS data (3243 children interviewed in the CDS supplement, for which it has been possible to recover information on their parents) and I identify the maximum number of items answered correctly at each age: in the age range 4-5 $J = 30$, in the age-range 6-8 $J = 50$ and finally, for $t = 9, 10, 11, 12, 13$ $J = 57$.

The CDS supplement provides several measures of child cognitive skills, based on the Woodcock Johnson Achievement Test Revised (WJ-R) (Woodcock and Johnson 1989). The outcome measure considered in this study is the Letter Word (LW) test, which is applied to all children older than four and proves a child's learning and reading skills (Hoffert et al. 1997). The raw LW score represents the sum of correct answers out of 57 items, ranging from 0 to 57. This measure is available in 1997, 2002 and 2007.

The CDS I (1997 wave) asks information to the primary caregiver on all non-parental child-care arrangements used for the child since childbirth; a set of follow-up questions was asked to the primary caregiver in the 2002 wave of the same supplement. Using both waves, I can recover the complete child-care history for the children interviewed in 1997. The variable of interests is the number of hours the child uses non-parental child care at each age. This variable refers to any type of child-care arrangement, either formal or informal, provided by people other than the parents.[19]

In 1997 and 2002, the Child Development Supplement includes another instrument to assess the time use of children: the Time Diary (TD). The TD is a unique feature of the CDS and consists in a chronological report filled out by the child or by the child's primary caregiver about the child's activities over a specified 24-hour period.[20] Each participating child completed two time diaries: one for a weekday (Monday-Friday) and one for a weekend day (Sunday or Saturday). The TD additionally collects information on the social context of the activity by specifying with whom the child was doing the activity and who else was present but not engaged. The variable weekly time with the mother is constructed by multiplying the daily hours the child spends with the mother by 5 for the weekday and by 2 for the weekend day, and summing up the total hours in a week.[21]

I take information on mothers and fathers linking the CDS data to the main PSID surveys. Since children in 1997 have different ages, ranging from 0 to 13 years old, in order to identify the necessary information for all children in any period defined by the model, CDS data should be matched with family information from PSID surveys in the years 1985-2007.[22] The family information I gather includes each parent's hours of work, wage and non labor income in each period.[23]

[19]The CDS questionnaire allows the primary caregiver to indicate more than one arrangement used at each age of the child. If the primary caregiver used simultaneously more than one arrangement in a period, I define the child care variable exploiting the information on the arrangement used more hours per week.

[20]The primary caregiver completed the time diary for the very young children (e.g., younger than 3), while older children and adolescents were expected to complete the time diaries themselves (ISR 2010a,b).

[21]The TD distinguishes between time spells when the child is with the mother only, time spells when the child is with the father only and time spells when the child is with both the mother and the father. The analysis has been performed using only time spells when the child is only with the mother, so that all remaining time spells indicate that the child is not receiving investments from the mother. In order to see whether the results are sensitive to this assumption, I re-estimate the model using an alternative definition of time investments, including also the time spells when the child is with both the father and the mother. Results are reported in Appendix D.1.

[22]For instance, to identify household information for all relevant periods for a child born in 1996 (1 year old in 1997) I need to use PSID surveys from 1997 to 2007; instead, if a child is born in 1986 (aged 11 years in 1997) I need to use PSID surveys from 1987 to 1999. All PSID surveys in the period 1985-2007 have been exploited, and the children included in the final sample were born between 1984 and 1996. See Appendix B, Tables B.1 and B.2.

[23]Between 1985 and 1997 PSID interviews were conducted annually but, since then, interviews have been biennial. Note that all the variables that I use from the main PSID surveys concerning labor and non

All relevant variables are constructed for each age of the child, defining age one as the first 12 months of child's life, age two as the next 12 months of the child's life, and so on. For the estimation of the model I consider all children without siblings interviewed in CDS I, living in intact households (where both mother and father are present for the entire period), without missing data on personal and parents' demographic characteristics and with at least one test score measure. The final sample is made up of 417 observations.

Table 1 shows the average values of all the variables for the period considered in the model. Mothers work, on average, 27 hours per week and use non-parental child care for almost 14 hours; moreover, they spend with their child, on average, 21 hours per week. The mothers' wages are on average 14 US$, while household income represents, on average, around 800 US$ per week. In the sample, the average raw score is around 35 out of 57. Figure 3 shows the distribution of the average test score measure by child's age.

The importance of the sample selection should be stressed and it should be considered what biases might be introduced into the analysis by focusing on the subsample of children in intact households without siblings. The sample selection, in fact, implies that all mothers' investments in child's ability are unrelated with the decision to marry or to cohabit and with fertility. However, if mothers in intact households have more marriage-oriented attitudes and unobservables determining their marriage/cohabitation decisions also influence their time allocation and fertility, they may be more likely to stay at home instead of working and to spend more time with their child. This may lead to an overestimation of the proportion of mothers not working or to an overestimation of mothers' preference for a child's ability. Similarly, mothers with only one child may have higher preferences for a child's ability and this may lead to an overestimation of the mother's use of the most productive investments. However, women in long-term relationships may also be more desirable in the labor market: if this is the case, this sample would be disproportionately represented by high productive mothers and may lead to overestimating the decision to work. Moreover, the fact of having only one child means that the mother has experienced only one work interruption as a result of childbirth, leading again to an overestimation of a mother's attachment to the labor market. Even though it is difficult to derive a unique direction of the bias induced by the sample selection, the arguments provided above suggest that it may oversample mothers who are more productive either in the labor market or at home with the child. This may provide an upper bound of the proportion of mothers in the labor force or of the productivity of a mother's time investments.[24]

5. Estimation

The model parameters are estimated using a Method of Simulated Moments estimator that minimizes the distance between several data statistics and their model counterparts. The data generating process implied by the model described in Section 3 allows to simulate the same statistics for the individuals (mothers and children) in the sample over the child's

labor income of the household members refer to the year before the survey. All monetary variables are deflated into 1997 US$ using the Consumer Price Index (CPI) History for the U.S. See Appendix B for further description of the data sources used for the analysis.

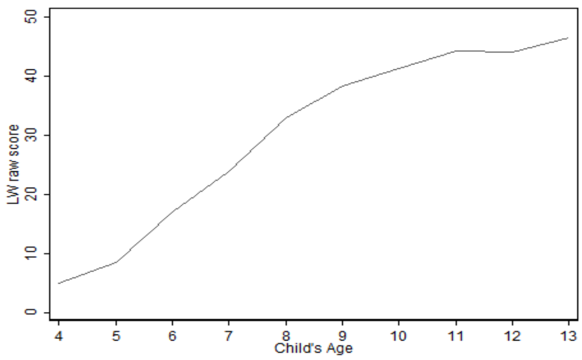
[24]Table B.4 in Appendix B compares the characteristics of the subsample used for the analysis ($N = 417$) with the ones of the entire PSID-CDS sample ($N = 3243$).

TABLE 1
Descriptive statistics on all variables for the entire period.

	Mean	SD	Min	Max
Child's LW raw score	35.10	14.47	1	57
Mother's hours of work	27.30	17.53	0	100
Proportion of working mothers	0.80	0.39	0	1
Non-parental child care hours	14.80	18.34	0	70
Mother's time with child	21.16	17.01	0.17	95.75
Child's gender: male	0.51	0.50	0	1
Child's birth weight	119.48	21.68	32	244
Mother's wage	14.37	10.27	5.01	133.93
Mother's age at child's birth	28.20	5.10	16	43
Mother's education	13.27	2.48	2	17
Mother's race: white	0.61	0.49	0	1
Father's education	13.30	2.47	1	17
Household income/10	79.14	64.41	0.01	883.49

NOTE. Monetary variables deflated into 1997 US$. Child's birth weight is expressed in ounces (88 ounces = 2500 grams). Household income includes father's labor income and household non labor income. Source: own elaboration from PSID-CDS data.

FIGURE 3
LW raw score by child's age.



NOTE. Source: own elaboration from PSID-CDS data.

life cycle. The full list of statistics used to construct the moment functions is reported in Table 2.

To recover the basic trends of data and, in particular, the observed differences in the mothers' time allocation, non-parental child care use and test scores across the ages of the child and between high and low educated mothers, I use as moments the average and standard deviation of the mothers choices and test scores by a child's age and the average mothers' choices by a mother's level of education. In particular, I compare mothers with more than a high school degree, who have, at the minimum, some college education to mothers with, at most, a high school degree.

TABLE 2
Statistics of actual and simulated data used for the estimation of the model.

Mother's choices
mean mother's hours of work, non-parental child care and mother's time with the child by child's age and by mother's education
proportion of mothers not working by mother's level of education
proportion of mothers not working or working more than 40 hours per week by child's age
proportion of mothers using non-parental child care less than 20 hours per week or more than 40 hours per week by child's age
proportion of mothers spending less than 30 hours per week with the child by child's age
Correlation among mother's choices and exogenous variables
corr mother's wage and mother's hours of work
corr household income and mother's hours of work
corr household income and non-parental child-care time
corr mother's hours of work and time with the child
corr mother's hours of work and non-parental child-care time
Test scores
mean test scores by child's age
Productivity parameters
OLS regression of test scores on child's age, maternal time with the child, mother's college education and their interactions (coefficients)
OLS regression of test scores on child's age, non-parental child-care time and its interactions with child's age (coefficients)
OLS regression of test scores on child's age, mother's wage and hours of work, household income and their interactions with child's age (coefficients)
Outcomes transition probabilities
prop of children with score in range p_y in years 1997 or 2002 and p_{y+5} in years 2002 or 2007
Wage equation and household income
mean, std deviation, median, 10th and 90th percentiles of mother's wage
OLS regression of mother's log wage on mother's education, age, age squared, race (coefficients)
mean of mother's wage conditional on mother's education, race and age
OLS regression of log household income on father's education, age, age squared, race (coefficients)
Mother's and child's unobservables
variance of the residuals from a mother's wage OLS reg on mother's education, age, age squared and race
variance of the residuals from a child's test score OLS reg on mother's and father's education, birth weight and gender
correlation between mother's wage residuals and child's test score residuals
correlation of mother's wage residuals in t with mother's wage residuals in $t-1$
correlation of child's test score residuals in t with child's test score residuals in $t-1$
Child's initial ability
OLS regression of test scores on child's age, mother's and father's education, child's birth weight and gender (coefficients)

NOTE. These statistics are computed using PSID-CDS data on children aged 0-12 in 1997, with at least one test score measure and without siblings, and simulated data according to the model defined in Section 3. Mother's time with the child is measured in 1997 and 2002; child's test scores are measured in 1997, 2002 and 2007; from 1997 on, mother's hours of work, mother's wage and household income are measured every two years and these variables refer to the year before the survey (see Section 4 and Appendix B for a description of the data). Household income includes both father labor income and household non labor income. Child's age t ranges from 1 to 13. Mother's and father's education are classified as continuous variables indicating years of education; mother's college education is a dummy variable equal to one if the mother has some college education; race is a dummy equal to 1 for being white; child's birthweight is a dummy equal to 1 for the child being less than 2500 grams at birth. Ranges p_y, with $y = 1997, 2002, 2007$ are defined according to the following ranges of the score distribution: $1st - 25th$ perc, $25th - 50th$ perc, $50th - 75th$ perc, $75th - 95th$ perc, higher than $95th$ perc.

To identify the mother's preference parameters and the trade-off between a mother's employment and child-care time, I use the correlation between a mother's choices and the exogenous variables in the model (i.e., a mother's wage and household income), as well as the correlation between a mother's labor supply and, respectively, non-parental child care and maternal child-care time. The productivity parameters in the CAPF are identified using the OLS coefficients of regressions where the dependent variable is the child's test score and the regressors are, respectively, the mother's choices in the previous period and their interactions with the age of the child. To identify the productivity parameters of a mother's child-care time and a mother's level of education, I also use the coefficients of the mother's level of education and the interaction between a mother's level of education and maternal child-care time. The parameter δ_4 representing the productivity of a child's ability in the previous period is recovered using transition probabilities from the first score measure available in the data (in 1997 or 2002) to the second score measure (available in 2002 or 2007).

To identify the distribution of a mother's and a child's unobservable skills, I use the variance of residuals from OLS regressions of log mother's wage and child's test scores, regressed, respectively, on a mother's age, education and race and on a child's birth weight, gender and parents' education; furthermore, I also exploit the correlation between the two, to identify the degree of correlation in the joint bivariate distribution. The variance of the residuals from the log wage regression may depend on the transitory shocks on wages (ϵ_t) or on the persistence of a mother's type productivity; similarly, the variance of the residuals from the test score regression may depend on either the measurement error of test score (v_t) or the persistence of a child's types. To disentangle the two, I also use as moments the correlation coefficient of each residual on the residual measured in the previous period: in other words, for both mothers and children, I correlate the level of residuals in each period on the residuals in the previous period, and I take this coefficient as the moment to be matched.

The identification of parameters in the wage equation is reached through the coefficients of a linear regression of log wage on a mother's education, age, age squared and race. Moreover, I use as moments the average mother's wage by the mother's educational level, age and race, and the 10th, 50th and 90th percentiles of the wage distribution. The parameters in the income process are identified using the average and standard deviation of income, as well as the coefficients of a OLS regression where the dependent variable is the log household income and the regressors are the father's age, his level of education and race. Finally, to identify the parameters in the initial level of a child's ability, I use the OLS regression coefficients of the mother's and the father's education and of the child's birth weight and gender on the child's test scores, also controlling for the child's age.

The simulation of the data is obtained by taking $N * R$ random draws from the initial distribution implied by the model, i.e., the child's and the mother's skills distributions, the mother's type preference distributions, and, for each period, from the wage and income distributions and from the distribution of the error in the test score measure.[25] After having drawn the child's level of ability, the wage offer and the level of income in the first period, the optimal choices of the mother are obtained by exploiting the optimal solutions derived in Section 3.2. This process is repeated for every period, up to the final one T. The simulated data are used to compute the same statistics defined in Table 2. Both actual and simulated statistics are used to construct the objective function to be minimized.

The Method of Simulated Moments estimator is then:

$$\hat{\theta} = arg\ min\ \hat{g}(\theta)'W\hat{g}(\theta) \tag{26}$$

where

$$\hat{g}(\theta) = \hat{m} - \hat{M}(\theta) \tag{27}$$

$\hat{m}$ is the vector of statistics defined from the actual data, while $\hat{M}(\theta)$ is the vector of simulated statistics according to the model. Given S number of moments, the weighting

[25] $N = 417$ and $R = 5$.

matrix is defined as:

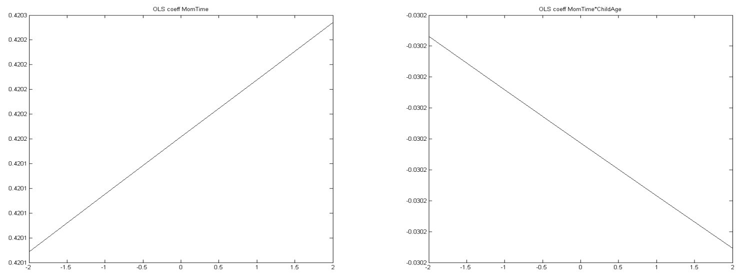
$$W = \begin{pmatrix} \hat{V}[\hat{m}_1]^{-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{V}[\hat{m}_S]^{-1} \end{pmatrix}$$

where $\hat{V}[\hat{m}]$ is estimated with non-parametric bootstrap. Appendix C provides further details on the estimation.

Identification of the model parameters relies on the choice of the statistics and moment conditions. Figures 4, 5 and 6 plot the values of the moment conditions used to identify the productivity parameters for maternal time with the child, the mother's education and non-parental child care and show how the moments vary by changing the values of the parameters, increasing the confidence in the choice of the statistics.

The estimation is done using the simplex algorithm, which is robust to non-smooth objective function. Identification of the model parameters also requires a unique solution for the minimization of the objective function defined by (26). In practice, it depends on the uniqueness of the minimum and on the curvature around it. I check that the objective function varies moving the values of the parameters, and results are reported in Figure C.1 in Appendix C.

FIGURE 4
Variation in moment conditions for mother's time productivity, perturbing the estimated parameter.

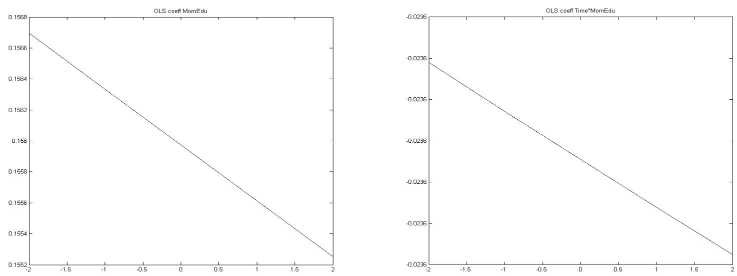


NOTES. This graph reports the values of the moment conditions obtained from a OLS regression of the child test score in period $t+1$ on maternal time with the child in period t and its interaction with the child's age t, perturbing the productivity parameter for maternal time with the child by 2 standard deviations up and down with respect to the estimated value.

6. Results

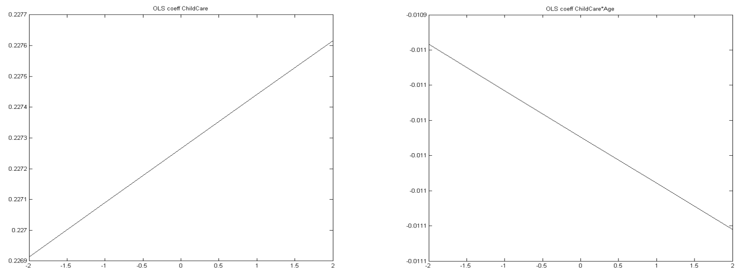
Table 3 reports the estimated parameters in the mother's utility function. Panel A shows the estimated values of the mother's utility for consumption and the child's ability for each mother's type, as well as the fraction of mothers in each group category: Type I corresponds to low levels of utility, while Type II corresponds to high levels. According to the estimated parameters, all mothers in the sample face a negative taste for consumption, while almost 70 percent of mothers have a negative taste for a child's ability; moreover, the utility from a child's ability for high type mothers almost doubles the level of utility for the low type. The parameters reported in Panel A are used to derive the αs parameters in the mother's utility function, according to the transformation specified by Equations

FIGURE 5
Variation in the moment conditions for the effect of maternal education on the productivity of mother's time with the child, perturbing the estimated parameter.



NOTES. This graph reports the values of the moment conditions obtained from a OLS regression of the child test score in period $t+1$ on maternal time with the child in period t, mother's education and their interactions, perturbing the productivity parameter for maternal education by 2 standard deviations up and down with respect to the estimated value.

FIGURE 6
Variation in the moment conditions for the productivity of non-parental child care, perturbing the estimated parameter.



NOTES. This graph reports the values of the moment conditions obtained from a OLS regression of the child test score in period $t+1$ on non-parental child care in period t, child's age t and their interactions, perturbing the productivity parameter for non-parental child care by 2 standard deviations up and down with respect to the estimated value.

(13), (14) and (15): Table 4 reports the results of this transformation, for each subgroup in the sample. To ease the interpretation of the coefficients, Figure 7 shows how these parameters relate to the transformed preference parameters for leisure, consumption and a child's ability. The three-dimensional feature of the graph also allows me to see how each preference parameter is correlated to the others, while Panel B of Table 4 reports the pairwise correlation coefficients among the preference parameters. Interestingly, the correlation between leisure and consumption is positive, while the correlation coefficients between the preference for a child's ability and the other goods are negative. Notice that the mother's preference for a child's ability can also be interpreted as her degree of altruism, and this can explain the estimated signs of the pairwise correlation coefficients. Moreover, the correlation coefficient between the preference for a child's ability and the preference for leisure is larger (in absolute value) than the correlation between the preference for a child's ability and the preference for consumption. This may suggest that mothers face a stronger trade-off between leisure and a child's ability than between a child's ability and consumption, though the final decisions in terms of time allocation and labor supply also depends on the estimated parameters in the wage equation and on the productivity parameters.

TABLE 3
Estimated untransformed parameters in the mother's utility function.

		Estimate	Std. Errors
Panel A. Mother's types			
γ_{2l}	Utility from consumption Type I	-2.1722	0.1637
γ_{2h}	Utility from consumption Type II	-0.6660	0.0441
γ_{3l}	Utility from child ability Type I	-3.1802	0.5062
γ_{3h}	Utility from child ability Type II	3.2866	0.2795
$\pi_{\gamma_{2l}}$	Proportion Type I consumption	0.8888	0.0210
$\pi_{\gamma_{2h}}$	Proportion Type II consumption	0.1112	(...)
$\pi_{\gamma_{3l}}$	Proportion Type I child ability	0.6828	0.0136
$\pi_{\gamma_{3h}}$	Proportion Type II child ability	0.3172	(...)
Panel B			
ρ	Weight on future child's ability in the last period	2.7747	0.8902
p	Hourly price of non-parental child care	19.1077	1.7928

NOTE. Standard errors are estimated with non-parametric bootstrap. See Appendix C.1 for further details. In Panel A, since type proportions should add to one, so that one of the type probabilities is obtained as a residuals, I do not report standard errors in this case.

TABLE 4
Estimated preference parameters in the mother's utility function.

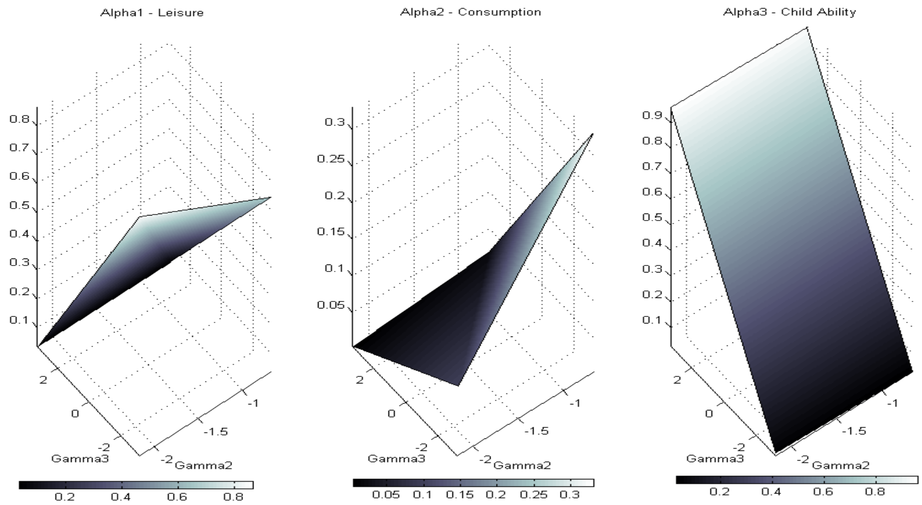
		Estimate	Std. Errors
Panel A. Preference Parameters			
$\alpha_{1\ \gamma_{2l}\gamma_{3h}}$	Preference for leisure (Type I consumption, Type II child ability)	0.0359	0.0167
$\alpha_{1\ \gamma_{2l}\gamma_{3l}}$	Preference for leisure (Type I consumption, Type I child ability)	0.8654	0.0211
$\alpha_{1\ \gamma_{2h}\gamma_{3h}}$	Preference for leisure (Type II consumption, Type II child ability)	0.0354	0.0157
$\alpha_{1\ \gamma_{2h}\gamma_{3l}}$	Preference for leisure (Type II consumption, Type I child ability)	0.6429	0.0124
$\alpha_{2\ \gamma_{2l}\gamma_{3h}}$	Preference for consumption (Type I consumption, Type II child ability)	0.0041	0.0015
$\alpha_{2\ \gamma_{2l}\gamma_{3l}}$	Preference for consumption (Type I consumption, Type I child ability)	0.0986	0.0137
$\alpha_{2\ \gamma_{2h}\gamma_{3h}}$	Preference for consumption (Type II consumption, type II child ability)	0.0182	0.0082
$\alpha_{2\ \gamma_{2h}\gamma_{3l}}$	Preference for consumption (Type II consumption, type I child ability)	0.3303	0.0116
$\alpha_{3\ \gamma_{2l}\gamma_{3h}}$	Preference for child ability (Type I consumption, Type II child ability)	0.9600	0.0179
$\alpha_{3\ \gamma_{2l}\gamma_{3l}}$	Preference for child ability (Type I consumption, Type I child ability)	0.0360	4.0564
$\alpha_{3\ \gamma_{2h}\gamma_{3h}}$	Preference for child ability (Type II consumption, Type II child ability)	0.9464	0.0017
$\alpha_{2\ \gamma_{2h}\gamma_{3l}}$	Preference for child ability (Type II consumption, Type I child ability)	0.0267	0.0147
Panel B. Pairwise correlation coeff. between preference parameters			
Corr(α_1,α_2)	Correlation pref. leisure and pref. consumption	+0.7288	
Corr(α_2,α_3)	Correlation pref. consumption and pref. child ability	−0.8011	
Corr(α_1,α_3)	Correlation pref. leisure and pref. child ability	−0.9937	

NOTE. Standard errors are estimated with non-parametric bootstrap. See Appendix C.1 for further details.

Table 5 shows the results from the wage equation and the income process. All parameters in the wage equation have the expected signs and reasonable magnitudes, though the coefficient for a mother's education is not statistically significant. The coefficients from the income equation indicate a positive correlation between a father's education and being white on the overall income available in the household, while the relationship between father's age and income is concave.[26]

[26]Mother's (father's) education is defined as years of education, while Race is a dummy variable indicating whether the mother (the father) is white.

Figure 7
Preference parameters as function of mother's utility from consumption (γ_2) and child's ability (γ_3).



NOTE. *Gamma2* and *Gamma3* stem for the parameters γ_2 and γ_3 representing the utility of the mother from consumption and the level of ability of the child, respectively. *Alpha1*, *Alpha2* and *Alpha3* represent the preference parameters for leisure (α_1), consumption (α_2) and the level of ability of the child (α_3), in the mother's utility function. The specification of the parameters α_1, α_2 and α_3 as a function of γ_2 and γ_3 is reported in Equations (13), (14) and (15).

Table 5
Estimated parameters for the wage and the income processes.

		Estimate	**Std. Errors**
Wage Equation			
μ_1	Coefficient of mother's years of education	0.0155	0.0089
μ_2	Coefficient of mother's age	0.1100	0.0024
μ_3	Coefficient of mother's age squared	-0.0016	0.0001
μ_4	Coefficient of mother's race	0.0927	0.0183
σ_ϵ	Std deviation wage shock	0.5291	0.0271
Household Income			
μ_{inc0}	Constant	-39.7588	11.6731
μ_{inc1}	Coefficient of father's years of education	0.1062	0.0205
μ_{inc2}	Coefficient of father's age	2.0427	0.6073
μ_{inc3}	Coefficient of fatehr's age squared	-0.0255	0.0076
μ_{inc4}	Coefficient of father's race	0.1840	0.1049
σ_{inc}	Std deviation income shock	0.0000	0.1763

NOTE. Standard errors are estimated with non-parametric bootstrap. See Appendix C.1 for further details.

As specified in Equation (17), a mother's wage depends also on mother's unobserved skills, which are assumed to be correlated with the child's skills entering the level of ability at birth. Table 6 reports the parameters identifying the mother's and the child's skills distribution. The skills level of high type mothers is 2.5 times that of the low skilled ones, while high skilled children have a skill level which is 20 percent larger than that of the low skilled. The proportion of low skilled mothers in the sample is equal to 17 percent, while almost half of children have a high level of skills at birth.

Table 7 presents the results of the parameters in the the initial level of ability and in the child's cognitive ability production function. The parameters shown in the first panel of this table represent the contributions of parents' education, child's birth weight and

Table 6
Mother's and child unobserved skills.

		Estimate	Std. Errors
μ_{mh}	Skill level for High Type mothers	0.4226	0.0620
μ_{ml}	Skill level for Low type mothers	0.1626	0.0323
ψ_{ch}	Skill level for High Type children	-3.9864	1.6543
ψ_{cl}	Skill level for Low Type children	-4.9141	1.6480
$\pi_{\mu_{cl\ ml}}$	Proportion Low Type children - Low Type mothers	0.1132	0.0203
$\pi_{\mu_{cl\ mh}}$	Proportion Low Type children - High Type mothers	0.3954	0.0843
$\pi_{\mu_{ch\ ml}}$	Proportion High Type children - Low Type mothers	0.0583	0.0113
$\pi_{\mu_{ch\ mh}}$	Proportion High Type children - High Type mothers	0.4330	0.0976

NOTE. Standard errors are estimated with non-parametric bootstrap. See Appendix C.1 for further details.

gender to the initial level of ability at birth. The identification of these parameters is problematic, because test scores can be observed only starting from age 4, and this is also confirmed by the fact that none of the estimated parameters is statistically different from zero.

Panel B of the same table shows the contribution of a mother's education to the productivity of a mother's time with the child, and the slopes of each input productivity with respect to a child's age. To simplify the presentation of the results, Figures 8 and 9 show the time-varying elasticities as a function of a child's age. Figure 8 reports the elasticities of child ability with respect to maternal time by a mother's level of education, and non-parental child-care time, while Figure 9 reports the elasticities with respect to expenditure for the child and the child's ability in the previous period. The first thing to notice is that the elasticity with respect to all inputs is higher during the early years and decreases over time, as suggested by previous studies on human capital accumulation (Carneiro and Heckman 2003; Heckman 2008).

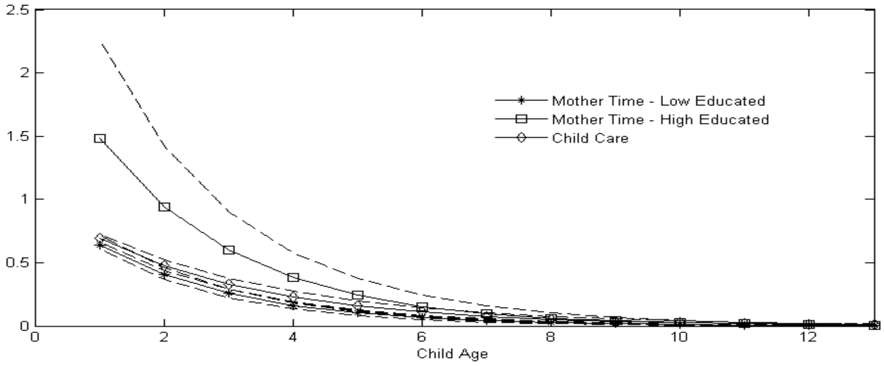
Table 7
Estimated parameters for the initial level of ability and the child's cognitive ability production function.

		Estimate	Std. Errors
Panel A. Child's initial ability parameters			
η_1	Contribution mother's education to child's endowment	1.0985	22.8435
η_2	Contribution father's education to child's endowment	11.7703	22.9557
η_3	Contribution birth weight to child's endowment	-10.1469	11.0214
η_4	Contribution gender to child's endowment	-8.4328	20.5536
Panel B. CAPF parameters			
ξ_{1edu}	Contribution of mother's education to mother's time productivity	0.8503	0.1952
$\xi_{1\tau}$	Slope productivity of mother's time with the child	-0.4544	0.0245
ξ_{2i}	Slope productivity of non-parental child care	-0.3675	0.0206
ξ_{3e}	Slope productivity of expenditure for the child	-0.8346	0.2142
ξ_{4A}	Slope productivity of child's ability in the previous period	-0.7417	0.1612
σ_v	Std deviation test score shock	0.9265	0.1809

NOTE. Standard errors are estimated with non-parametric bootstrap. See Appendix C.1 for further details.

According to Figure 8, maternal time with the child is at least as productive as non-parental child care, at any age of the child. Moreover, the productivity of a mother's child-care time substantially differs between low and college-educated mothers, especially during the child's first years of life. The elasticity of a child's ability with respect to maternal time for low educated mothers ranges between 0.55 when the child is one year old and almost zero when the child is 13; the productivity of maternal child-care time for

Figure 8
Elasticity of child's ability with respect to mother's time with the child and non-parental child care.



NOTE. This graph represents the productivity parameters for maternal child-care time (τ_t) and non-parental child care (i_t), as a function of mother's education and child's age $t = 1, 2, 3, \ldots 13$. *High educated* mothers indicate mothers with some college education, while *Low educated* mothers indicate mothers without a high school degree. The dashed lines represent the 95percent confidence interval for each parameter. The specification of the parameters is reported in Equations (19) and (20).

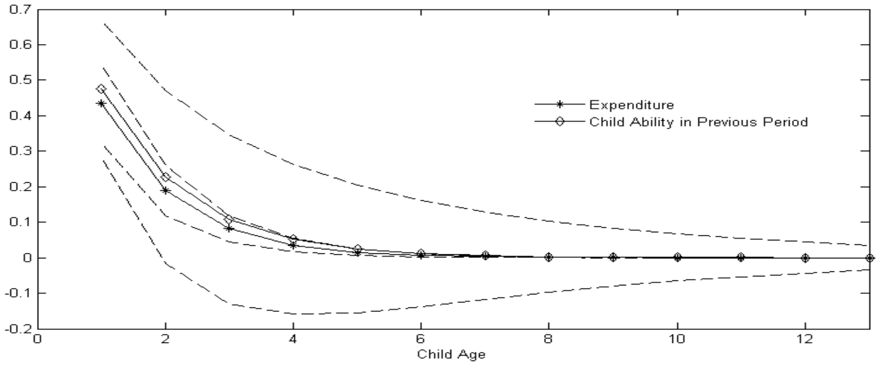
college-educated mothers, instead, reaches 1.5 when the child is aged one, but, similarly, decreases over time. This implies that, when the child is one year old, one percent increase in the mother's time leads to an increase in the level of ability of the child of 0.55 percent for low educated mothers, and 1.5 percent for high educated mothers; this roughly corresponds to an increase of a 4.6 percent of a standard deviation of the score distribution for low educated mothers and of a 10 percent of a standard deviation for the high educated. Interestingly, the productivity of non-parental child care is not statistically different from the productivity of the maternal child-care time of low educated mothers at any child's age; starting from age five, the three inputs do not show statistically different effects, so that non-parental child care becomes as productive as maternal child-care time for all types of mothers.

The fact that the elasticity of all inputs decreases over time may be due to other inputs, such as schooling, which play a role from age six onward.[27] Moreover, the decline in non-parental child care productivity when the child starts going to kindergarten or primary school can be explained by the different purposes that non-parental child care may have from the mother's point of view. In fact, the mother may choose a positive amount of child care if she works and needs someone looking after the child, but also if she thinks it can represent an input for the child's subsequent development. The educational role of child care can be less important when the child starts going to school, because he is receiving educational inputs from other institutions (i.e., schools), so that from this age on the custodial role can prevail. As a consequence, the productivity of non-parental child care might decrease over time even if the amount of time that is used remained constant.

[27]Appendix D.2 shows the results from two sensitivity analyses performed with respect to schooling: the first one re-estimates the model including in the definition of non-parental child care also a fixed amount of time when the child attends primary school (5 hours per day); the second, instead, estimates a different version of the model where schooling is incorporated as a total factor productivity in the production function for child's ability.

Figure 9 shows the elasticity of child ability with respect to the expenditure in goods for the child and the child's ability in the previous period. The result for expenditure seems in line with existing literature saying that economic conditions in early childhood are more important for children's cognitive outcomes than those during adolescence (Duncan and Brooks-Gunn 1997; Duncan et al. 1998; Levy and Duncan 2012): in fact, the estimated coefficient for expenditure is statistically different from zero only before age six. Similarly, the estimated parameter for the elasticity of a child's ability with respect to the level of ability in the previous period is statistically significant only in the first years of the child's life, suggesting that the model is not capturing any persistence in the development of the child's ability.

FIGURE 9
Elasticity of child's ability with respect to expenditure and the level of ability of the child in the previous period.



NOTE. This graph represents the productivity parameters for expenditure in goods bought for the child (e_t) and the level of ability of the child in the previous period (A_t), as a function of child's age $t = 1, 2, 3, \ldots 13$. The dashed lines represent the 95percent confidence interval for each parameter. The specification of the parameters is reported in Equations (21) and (22).

The estimated parameters reported in Figures 8 and 9 shed light on how the different productivity of inputs affects the mother's decision-making process. In fact, the use of non-parental child care may have different effects on the children in the sample because of the different home alternative to which children are exposed. Since mothers know the relative productivity of their time investments with respect to that provided by the alternative forms of care, college-educated mothers are aware of the lower productivity of non-parental child care with respect to theirs. Hence, their final decision of whether to join the labor force depends on whether the alternative inputs (i.e., non-parental child care and expenditure) can compensate for the reduction in the mother's child-care time. College-educated mothers may thus find it more profitable to spend more time with their child instead of working and using non-parental child care, because of the higher productivity of their time with respect to that of the alternative forms of care. This may explain the recent evidence of highly educated women exiting the labor force to care for their children at higher rates than their less educated counterparts.[28]

[28]This trend has been reported and analyzed, for instance, by Juhn and Potter (2006) and Macunovich (2010).

6.1. ***Goodness of fit of the model.*** Table 8 shows the fit of the model for the mother's choice variables. The overall fit of the model for the mother's choices is good, though the model slightly overestimates the mother's labor supply and the amount of non-parental child care. As previously discussed, this could be due to one of the assumptions needed to estimate the model, which is the exogeneity of the wage process with respect to the mother's decisions, implying that the mother does not face any costs associated with her absence from the labor market. Moreover, the overestimation of non-parental child care can be determined by the absence of a schooling investment in the child development process specified by the model.

Table 8
Goodness of fit for mother's choices.

	Actual data	**Simulated data**
Mother's hours of work	27.3032	36.6144
Non-parental child care	14.7968	17.8419
Mother's time with the child	21.1600	25.2382

NOTE. Actual data represent PSID-CDS data on children aged 0-12 in 1997, with at least one test score measure and without siblings. See Section 4 and Appendix B for further details on the data. Simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values.

Table 9 shows how the model performs in fitting the data concerning the wage and the income processes. Specifically, it shows the average and standard deviation of wage and income, observed in the actual and in the simulated data. The model predicts well the average wage and income and there are no differences between the actual and simulated data concerning the standard deviation of income. Figure 10 shows the model fit for the child's score measure. Despite failing to predict the pattern of test scores in the child's first years of life, the model predicts quite well the test score measures for the child's subsequent years.

Table 9
Goodness of fit for mother's wage and household income.

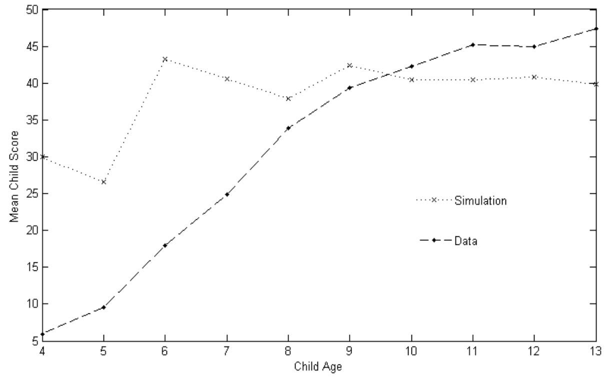
	Actual data	**Simulated data**
Mean mother's wage	14.3659	13.023
Std mother's wage	10.2725	7.6254
Mean household income	7.9136	7.9201
Std household income	6.4406	6.4406

NOTE. Actual data represent PSID-CDS data on children aged 0-12 in 1997, with at least one test score measure and without siblings. See Section 4 and Appendix B for further details on the data. Simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values.

7. Policy simulations

In this section, I use the estimated model to simulate the effects of maternity leave policies aimed at increasing the amount of time the mother spends with the child. These policies are explicitly aimed at favoring mother-child interactions during the first years of the child's life by delaying the mother's re-entry into the labor force. While in the U.S.

Figure 10
Goodness of fit for child's test score measure by child's age.



NOTE. Actual data represent PSID-CDS data on children aged 0-12 in 1997, with at least one test score measure and without siblings. See Section 4 and Appendix B for further details on the data. Simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values.

maternity leave is unpaid and not guaranteed to all working mothers, some European countries, such as Germany, Sweden and Norway, offer very generous policies, where the leave can last for two years and be sustained by a payment scheme; moreover, countries like Germany and Italy explicitly provide compulsory leave periods when mothers cannot work (Ray et al. 2009). Previous studies looking at the effects of maternal employment on children's subsequent development find substantial negative effects and suggest that a reduction in maternal working time could be beneficial for children's development (Bernal 2008; Ermish and Francesconi 2013). However, a recent body of literature assessing the effects of maternity leave policies on the subsequent development of the new-born child provides mixed results. Some studies find positive effects of longer paid maternity leave on children's final education, arguing that the beneficial effect depends on the longer period of time spent by the mother with the child (Carneiro et al. 2014); others, instead, suggest that longer paid maternity leave does not have any effects on children's subsequent cognitive development, while the unpaid type can be harmful because it decreases the available income in the household (Dustmann and Schönberg 2011). In order to understand the effects of such a policy and the role played by each component, i.e., mother's child-care time and payment, I simulate the effects of three leave policies, mandating the mother not to work the first two years of the child's life: the first is completely unpaid, the second provides a lump-sum payment equal to 280$ per week, while the third mandates a payment which is proportional to the wage the mother would have received if working.[29]

Table 10 reports the changes in a mother's choices after the implementation of the policies at different ages of the child. The first thing to notice is that the payment aspect of the policy does not affect the mother's time allocation, but it strongly affects the level of expenditure and non-parental child care use. This is because, in the simulated scenario, the mother is not allowed to choose whether to re-enter the labor market before the child

[29]More precisely, the lump-sum payment is defined as being equal to the median wage in the sample multiplied by the average working time observed in the data (23 hours per week); the wage-proportional payment is, instead, equal to the product between a mother's wage offer and the average working time of mothers (23 hours per week).

Table 10
Simulation of maternity leave policies. Effects on mother's choices at different child's ages.

		Baseline	**(a) Unpaid**	**(b) Lump-sum pay**	**(c) Wage-prop pay**
	Child age<= 2	44.74	-100.00	-100.00	-100.00
Mother's hours of work	Child age<= 5	42.52	-42.09	-42.09	-42.09
	Child age<= 12	36.40	-18.91	-18.91	-18.91
	Child age<= 2	36.99	101.19	101.19	101.19
Mother's time with the child	Child age<= 5	35.75	41.89	41.89	41.89
	Child age<= 12	25.21	22.85	22.85	22.85
	Child age<= 2	30.26	24.15	24.15	24.15
Mother's leisure	Child age<= 5	33.73	8.67	8.67	8.67
	Child age<= 12	50.39	2.23	2.23	2.23
	Child age<= 2	18.44	-99.30	-62.22	-56.11
Non-parental child care	Child age<= 5	20.30	-36.07	-22.60	-20.38
	Child age<= 12	17.81	-15.82	-9.91	-8.94
	Child age<= 2	177.53	-99.31	-62.25	-56.16
Expenditure	Child age<= 5	111.58	-63.20	-39.61	-35.74
	Child age<= 12	47.54	-57.05	-35.76	-32.26

NOTE. This table reports percentage changes with respect to the baseline levels after the implementation of (a) a policy mandating mothers' unpaid leave from work in the first two years of the child's life, (b) a policy mandating mothers' leave from work in the first two years of the child's life and a lump-sum payment in the same period of 280$ per week, and (c) a policy mandating mothers' leave from work in the first two years of the child's life and a payment in the same period proportional to a mother's wage offer.

becomes two years old, and, hence, the payment only affects the willingness of the mother to spend for the child, either buying goods or non-parental child care services. In the case of an unpaid leave, during the first two years of the child's life, the absence of payment and of the mother's labor income determines a strong decline in the amount of money spent by the mother for the child (99 percent decline for both expenditure and non-parental child care), while the lump-sum and the wage-proportional payments lead to a 62 percent and 56 percent reduction, respectively. It should also be noticed that the policy paying proportionally to a mother's wage leads to a smaller decline in expenditure and non-parental child care investments than that paying lump-sum. This effect is most likely to be driven by mothers in the top part of the wage distribution, who find the wage payment more profitable than the lump-sum payment, with respect to the baseline case where they can work. Finally, in the case of the paid leaves, non-parental child care declines by a smaller amount than expenditure: this can be explained by the higher productivity of the former than the latter, at any child's age.

Table 11 shows how the mother's time allocation in the first two years of the child's life changes by a mother's level of education. During the first two years of the child's life, the policy mandates mothers not to work and high and low educated mothers allocate differently their time between child care and leisure. The figures in the table represent the fraction of the decline in labor supply allocated to either child care or leisure. While both high and low educated mothers allocate more than 80 percent of their time to child care, the high educated give priority to the time they spend with the child and have a lower amount of leisure. In other words, college-educated mothers allocate a larger share of their gained time to child care than the low educated do, while this relationship for leisure is reversed. These results show not only that a mother's child-care time does not entirely correspond to a mother's time out of work, but also that the heterogeneous productivity in the child development process leads to a different time allocation across mothers' types.

Table 11
Simulation of maternity leave policies. Effects on mother's time allocation in the first two years of the child life.

		(a) Unpaid	(b) Lump-sum pay	(c) Wage-prop pay
Mother's time with the child	All sample	83.67	83.67	83.67
	College educated	84.57	84.57	84.57
	Low educated	82.95	82.95	82.95
Mother's leisure	All sample	16.33	16.33	16.33
	College educated	15.43	15.43	15.43
	Low educated	17.05	17.05	17.05

NOTE. This table reports the variation in mother's child care and leisure time as percentages of the total decline in mother's labor supply in the first two years of the child's life, after the implementation of (a) a policy mandating mothers' unpaid leave from work in the first two years of the child's life, (b) a policy mandating mothers' leave from work in the first two years of the child's life and a lump-sum payment in the same period of 280$ per week, and (c) a policy mandating mothers' leave from work in the first two years of the child's life and a payment in the same period proportional to a mother's wage offer. *College educated* indicates mothers with some college education, while *Low educated* indicates mothers without a high school degree.

Table 12 shows the effects of the simulations on the child's test scores, measured at age five and at age twelve: this distinction allows me to see potentially different effects over the child's life cycle. Interestingly, the unpaid leave has a short-term negative effect on the child's test scores, while the effects of the paid ones are positive. The unpaid leave policy determines a 20 percent decline in the child's test score at age 5, which corresponds to a decline of 0.4 percent of a standard deviation of the score distribution; moreover, the negative effect is larger for children with low educated mothers than for the children of the high educated. Despite the increase in maternal child-care time, the policy also determines a decline in the other investments the mother can make on the child's ability, i.e, non-parental child care and expenditure, yielding the negative effect on test scores observed when the child is five years old. Furthermore, the different productivity of maternal child-care time across mother's level of education contributes to the different effects found at this age, leading to a smaller negative effect (in absolute value) for high educated mothers than for their less educated counterparts. The payment component of the other two simulated policies seem to compensate for the negative effects induced by the unpaid leave, yielding a 2 percent increase in the child's test scores at age 5. Interestingly, the lump-sum payment policy provides the same positive effect for all children, regardless of their mothers' level of education, while the effects of the policy where the payment depends on a mother's wage is different across a mother's level of education: the effect is 0.4 percent of a standard deviation for the children of a high educated mother and 0.3 percent of a standard deviation for the children of a low educated mother. Hence, the results of these simulation exercises suggest that the lump-sum payment can be preferred to the wage-proportional one for redistributive purposes.

When observing the child's test score at age twelve, there are no effects of any of the simulated policies. In the ages following the implementation of the policy, mothers react to the changes in their time allocation imposed by the policy, by increasing the other inputs in the child development process, i.e., non-parental child care and expenditure for the child, which have faced a sharp decline following the absence of the mothers' labor income. Interestingly, the reduction in non-parental child care with respect to the baseline scenario is smaller than that in expenditure, and this could be explained by the higher productivity of the former with respect to the latter. Moreover, the difference in the effects on the test scores between high and low educated mothers progressively fades out because the difference in a mother's time productivity across a mother's level of education disappears as the child ages. This may explain the zero effect found on the test scores at age twelve and may support the recent results in the literature showing that maternity leave policies do not have any effect on the long-term educational outcomes of children (Dustmann and Schönberg 2011).

Table 12
Simulation of maternity leave policies. Effects on the test scores at different child's ages.

		Baseline	**(a) Unpaid**	**(b) Lump-sum pay**	**(c) Wage-prop pay**
All sample	Child age= 5	26.57	-0.238	0.018	0.019
	Child age= 12	39.93	0.000	0.000	0.000
College educated	Child age= 5	27.44	-0.200	0.018	0.020
	Child age= 12	40.05	0.000	0.000	0.000
Low educated	Child age= 5	25.14	-0.306	0.018	0.016
	Child age= 12	39.79	0.000	0.000	0.000

NOTE. This table reports percentage changes in test scores with respect to the baseline levels after the implementation of (a) a policy mandating mothers' unpaid leave from work in the first two years of the child's life, (b) a policy mandating mothers' leave from work in the first two years of the child's life and a lump-sum payment in the same period of 280$ per week, and (c) a policy mandating mothers' leave from work in the first two years of the child's life and a payment in the same period proportional to a mother's wage offer. *College educated* indicates mothers with some college education, while *Low educated* indicates mothers without a high school degree.

8. Concluding remarks

This paper estimates a behavioral model where labor supply, non-parental child care, expenditure for the child and time allocation choices of the mother are considered endogenous. In contrast to existing studies, I take into account the additional choice the mother makes concerning her time allocation between leisure and time with the child. Maternal time and non-parental child care serve as inputs in a child's development process that represents a constraint to the mother's utility maximization problem.

In line with previous studies on human capital accumulation reporting diverse productivity of investments over time (Heckman 2008), the results show that the productivity of both maternal child-care time and non-parental child care decreases as the child ages. Moreover, the elasticity of child's ability with respect to maternal child-care time substantially differs by a mother's level of education: child-care time of mothers with some college education is more productive than that of low educated mothers, in particular when the child is aged less than six. When the child is one year old, one percent increase in child-care time for college-educated mothers increases a child's ability by 1.5 percent, corresponding to 10 percent of a standard deviation; in contrast, one percent increase in child-care time for low educated mothers increases child's ability by 0.55 percent, corresponding to 4.6 percent of a standard deviation of the score distribution. This result is consistent with a framework where the mother's level of education is positively correlated with educational-enhancing activities, such as reading, talking and playing with the child, which seem to be effective for the subsequent cognitive development of the child (Kalb and Van Ours 2014; Price 2010). Moreover, maternal time with the child is found to be at least as productive as non-parental child care: the productivity of child-care time for college-educated mothers is higher than that of non-parental child care, while the productivity of child-care time of low-educated mothers is not statistically different from that of non-parental child care.

These results imply that a mother's employment can be detrimental for the subsequent development of the child if the productivity of maternal child-care time is higher than that of non-parental child care. The mother may also compensate for the reduction in maternal time by increasing the expenditure in goods bought for the child. However, the estimated productivity of expenditure is lower, at any child's age and for any level of mothers' education, than that of maternal child-care time. The negative effects of maternal employment are potentially larger for high educated mothers than for the low educated: hence, mothers with some college education may find it profitable to decrease their labor supply in order to stay home with the child, because they are aware of the lower productivity of the alternative form of care with respect to theirs.

The estimated model is used to simulate the effects of leave policies, mandating mothers not to work in the first two years after childbirth. The results show that the heterogeneous productivity of mother's child-care time by a mother's level of education leads to a diverse allocation of time during the leave, when the mother cannot work. More precisely, mothers do not entirely allocate their time out of work to child-care, and college-educated mothers allocate a larger fraction of their time to child-care than the low educated do. Moreover, the policies have different effect on the mother's decision to buy goods and services (i.e., non-parental child care) for the child, depending on whether the leave is paid or not. In case of the unpaid leave, the mother not only eliminates the use of non-parental child care, but also the expenditure in goods for the child. Instead, in a paid leave policy scheme, in which the payment is proportional to the mother's wage, high educated mothers increase their expenditure for the child and non-parental child care use more than the low educated do. The effects of the policies on the child's test score at age five are positive or negative, depending on whether the leave is paid or not, which suggests a positive role played by the expenditure for the child and non-parental child care in the first two years of the child's life too. The negative effect of the unpaid leave policy is larger (in absolute value) for low educated mothers than for the college-educated, not only because the former have a less productive child-care time, but also because they allocate a smaller fraction of their time out of work to child care. The positive effect of the paid leave policy, in case of payment proportional to the mother's wage, is instead larger for college-educated mothers than for the low educated. Hence, such a policy would exacerbate the inequality in children's achievements by a mother's level of education. The results of the policy simulations do not reveal any effect on the child's test score at age 12: this is because the mother, after the leave, readjusts her investment decisions and labor supply, up to the levels of the baseline scenario. This result seems to support the findings of studies assessing the effects of maternity leave policies and reporting null effects in the long run (Baker and Milligan 2010; Dustmann and Schönberg 2011).

This study provides two relevant contributions to the research on the effects of maternal and non-parental child care on the child cognitive development. First, it highlights the importance of considering the mother's time allocation choice between child-care time and leisure. The paper shows that the mothers may not entirely allocate their time out of work to child care and that this has implications for the effects of policies aimed at increasing the amount of time they spend with their child, such as maternity leave. Second, the results of the paper show how a mother's decisions are affected by the relative productivity of maternal child-care time with respect to non-parental child care. Nonetheless, the analysis leaves space for further research. For instance, the model does not distinguish between different kinds of child care and assumes that any type of non-parental care has the same productivity for child development. Moreover, little is known about the substitutability or complementarity of mother's child-care time and non-parental child care in the production for cognitive achievement. Future research should better understand how the mother's investment decisions could change, varying the quality of the alternative forms of care, and how these interact in the production function for child's cognitive ability.

References

Altonji, J. G. and L. M. Segal (1996). Small-sample bias in GMM estimation of covariance structures. *Journal of Business & Economic Statistics 14*(3), 353–366.

Baker, M. and K. Milligan (2010). Evidence from maternity leave expansions of the impact of maternal care on early child development. *Journal of Human Resources 45*(1).

Baydar, N. and J. Brooks-Gunn (1991). Effects of maternal employment and child care arrangements on preschoolers' cognitive and behavioral outcomes. *Developmental Psychology 27*, 932–945.

Becker, G. S. and N. Tomes (1986). Human capital and the rise and fall of families. *Journal of Labor Economics 4*, S1–S39.

Belsky, J. and D. Eggebeen (1991). Early and extensive maternal employment and young childrens socioemotional development: Children of the National Longitudinal Survey of Youth. *Journal of Marriage and the Family 53*, 10831110.

Bernal, R. (2008). The effect of maternal employment and child care on children's cognitive development. *International Economic Review 49*(4), 1173–1209.

Bernal, R. and M. P. Keane (2011). Child care choices and children's cognitive achievement: The case of single mothers. *Journal of Labor Economics 29*(3), 459–512.

Bianchi, S. M. (2000). Maternal employment and time with children: dramatic change or surprising continuity? *Demography 37*(4), 401–414.

Blau, D. M. and J. Currie (2006). Pre-school, day care, and after-school care: Who's minding the kids? In E. A. Hanushek and F. Welch (Eds.), *Handbook of The Economics of Education*, Volume 2.

Blundell, R., L. Pistaferri, and I. Preston (2008). Consumption inequality and partial insurance. *American Economic Review 98*(5), 1887 – 1921.

Cameron, A. C. and P. K. Trivedi (2005). *Microeconometrics. Methods and applications.* New York, USA: Cambridge University Press.

Carneiro, P. and J. J. Heckman (2003). Human capital policy. In J. J. Heckman, A. B. Krueger, and B. M. Friedman (Eds.), *Inequality in America: What Role for Human Capital Policies?*, pp. 77–239. Cambridge: MA: MIT Press.

Carneiro, P., K. Løken, and K. Salvanes (2014). A flying start? Maternity leave benefits and long run outcomes of children. *Journal of Political Economy Forthcoming.*

Chase-Lansdale, P. L., R. A. Moffit, B. J. Lohman, A. J. Cherlin, R. L. Coley, L. D. Pittman, J. Roff, and E. Votruba-Drzal (2003). Mother's transitions from welfare to work and the well-being of preschoolers and adolescents. *Science 299*, 1548–1552.

Cunha, F., J. J. Heckman, L. J. Lochner, and D. V. Masterov (2006). Interpreting the evidence on life cycle skill formation. In E. A. Hanushek and F. Welch (Eds.), *Handbook of the Economics of Education*, pp. 697–812. Amsterdam: North-Holland.

Cunha, F., J. J. Heckman, and S. M. Schennach (2010). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica 78*(3), 883–931.

Davidson, R. and J. G. MacKinnon (2003). *Econometric theory and method.* New York: Oxford University Press.

Del Boca, D., C. J. Flinn, and M. Wiswall (2014). Household choices and child development. *Review of Economic Studies 81*(1), 137–185.

Del Boca, D., C. Monfardini, and C. Nicoletti (2012). Self investments of adolescents and their cognitive development. Collegio Carlo Alberto Notebooks N.265.

Desai, S., L. P. Chase-Lansdale, and R. T. Michael (1989). Mother or market? Effects of maternal employment on the intellectual ability of 4-year old children. *Demography 26*, 545–561.

Duncan, G. J. and J. Brooks-Gunn (1997). Income effects across the life span : integration and interpretation. In G. J. Duncan and J. Brooks-Gunn (Eds.), *Consequences of growing up poor*. New York: Russel Sage Foundation.

Duncan, G. J., W. J. Yeung, J. Brooks-Gunn, and J. R. Smith (1998). How much does childhood poverty affect the life chances of children? *American Sociological Review 63*(3), 406–423.

Dustmann, C. and U. Schönberg (2011). Expansions in maternity leave coverage and childrens long-term outcomes. *American Economic Journal: Applied Economics 4*(3), 190–224.

Ermisch, J. and M. Francesconi (2005). Parental employment and childrens welfare. In T. Boeri, D. D. Boca, and C. Pissarides (Eds.), *Women at Work: an Economic Perspective*. Oxford University Press.

Ermish, J. and M. Francesconi (2013). The effect of parental employment on child schooling. *Journal of Applied Econometrics 28*(5), 796–822.

Guryan, J., E. Hurst, and M. Kearney (2008). Parental education and parental time with children. *Journal of Economic Perspectives 22*(3), 23–46.

Hale, L., L. M. Berger, M. K. LeBourgeois, and J. Brooks-Gunn (2011). A longitudinal study of preschoolers'language-based bedtime routines, sleep duration, adn well-being. *Journal of Family Psychology 25*, 423–433.

Heckman, J. J. (2007). The economics, technology, and neuroscience of human capability formation. *PNAS 104*(33), 13250–13255. Proceedings of the National Academy of Sciences of the United States of America.

Heckman, J. J. (2008). Schools, skills, and synapses. *Economic Inquiries 46*(3), 289.

Heckman, J. J. and B. Singer (1984). A method for minimizing the impact of distributional assumptions in econometric models for duration data. *Econometrica 52*(2), 271–320.

Hoffert, S. L., P. E. Davis-Kean, J. Davis, and J. Finkelstein (1997). *The Child Development Supplement to the Panel Study of Income Dynamics. The 1997 User Guide*. Ann Arbor, MI: Survey Research Center, Institute for Social Research, The University of Michigan.

Hoffert, S. L. and J. F. Sandberg (2001). How American children spend their time? *Journal of Marriage and the Family 63*, 295–308.

ISR (2010a). *The Panel Study of Income Dynamics Child Development Supplement. The User Guide Supplement for CDS-I*. Ann Arbor, MI: Survey Research Center, Institute for Social Research, The University of Michigan.

ISR (2010b). *The Panel Study of Income Dynamics Child Development Supplement. The User Guide Supplement for CDS-II*. Ann Arbor, MI: Survey Research Center, Institute for Social Research, The University of Michigan.

James-Burdumy, S. (2005). The effects of maternal labor force participation on child development. *Journal of Labor Economics 23*(1), 177–211.

Juhn, C. and S. Potter (2006). Changes in labor force participation in the United States. *Journal of Economic Perspectives 20*(3), 27–46.

Kalb, G. and J. C. Van Ours (2014). Reading to young children: A head-start in life? *Economics of Education Review 40*, 1–24.

Keane, M. P. and R. A. Moffitt (1998). A structural model of multiple welfare program participation and labor supply. *International Economic Review 39*(3), 553–589.

Leibowitz, A. (1974). Home investments in children. In T. W. Schultz (Ed.), *Marriage, Family, Human Capital, and Fertility.* NBER. National Bureau of Economic Research.

Leibowitz, A. (1977). Parental inputs and children's achievement. *Journal of Human Resources 12*(2), 242–251.

Levy, D. and G. J. Duncan (2012). Using siblings samples to assess the effect of childhood family income on completed schooling. Northwestern University Working Paper.

Loeb, S., M. Bridges, D. Bassok, B. Fuller, and R. W. Rumberger (2007). How much is too much? The influence of preschool centers on children's social and cognitive development. *Economics of Education Review. Economics of Early Childhood Education Issue 26*, 52–66.

Macunovich, D. J. (2010). Reversals in the patterns of womens labor supply in the United States,19772009. *Monthly Labor Review 133*(11), 16–36.

Magnuson, K. A., C. J. Ruhm, and J. Waldfogel (2007). Does prekindergarten improve school preparation and performance? *Economics of Education Review. Economics of Early Childhood Education Issue 26*, 33–51.

Mroz, T., H. Liu, and W. Van der Klaauw (2010). Maternal employment, migration and child development. *Journal of Econometrics 156*(1), 212–228.

Novick, M. R. (1966). The axioms and principal results of classical test theory. *Journal of Mathematical Psychology 3*, 1–18.

Parcel, T. L. and E. G. Menaghan (1994). Early parental work, family social capital, and early childhood outcomes. *The American Journal of Sociology 99*(4), 972–1009.

Price, J. (2010). The effects of parental time investments: Evidence from natural within-family variation. mimeo, Brigham Young University.

Raikes, H., B. Pan, G. Luze, C. Tamis-LeMonda, J. Brooks-Gunn, J. Constantine, L. Tarullo, H. Raikes, and E. Rodriguez (2006). Mother-child bookreading in low-income families: Correlates and outcomes during the first three years of life. *Child Development 77*, 924–953.

Ray, R., J. Gornick, and J. Schmitt (2009). Maternity leave policies in 21 countries: assessing generosity and gender equality. Centre for Economic and Policy Research (CEPR), Washington, D.C., USA.

Ribar, D. C. (1992). Child care and the labor supply of married women. Reduced form evidence. *Journal of Human Resources 27*(1), 134–165.

Rossin-Slater, M. (2011). The effects of maternity leave on children's birth and infant health outcomes in the United States. *Journal of Health Economics 30*(2), 221–239.

Ruhm, C. J. (2004). Parental employment and child cognitive development. *Journal of Human Resources XXXIX*, 155–192.

Todd, P. and K. Wolpin (2003). On the specification and estimation of the production function for cognitive achievement. *The Economic Journal 113*, F3–F33.

Train, K. (2009). *Discrete Choice Methods with Simulation* (2 ed.). Cambridge University Press.

U.S. Census Bureau (2000). *Statistical Abstract of the United States: 2000.*

U.S. Census Bureau (2013). American Time Use Survey 2005-2009. http://www.bls.gov/tus/.

Woodcock, R. W. and M. E. B. Johnson (1989). *Tests of Achievement, Standard Battery [Form B].* Chicago, IL: Riverside Publishing.

Wooldridge, J. (2002). *Econometric Analysis of Cross Section and Panel Data.* Cambridge: MA: MIT Press.

Appendix A. Analytic solution of the model

In this Appendix I derive analytically the closed-form solutions of the model, for all the choice variables. The process of backward induction involves the solution of the optimization problem in each period, starting from the last one, T. Consider first the choice variables i_t, e_t and τ_t. The first step is to find the optimal child care, expenditure and time input decisions at time T. The value function of the mother at period T can be written as:

$$V_T = max_{\tau_T, i_T, e_T}\ \alpha_1 ln(TT - h_T - \tau_T) + \alpha_2 ln(w_T h_T + I_T - pi_T - e_t) + \alpha_3 ln(A_T) + \\ + E_T \beta\{\tilde{V}_{T+1} + \rho\alpha_3 ln A_{T+1}\} \tag{A.1}$$

where the variables l_T and c_T have been already substituted using the time and budget constraints, and the braces include the terminal period value function, as specified in (6).

The optimal solutions for τ_T^c, i_T^c and e_T^c at period T, conditional on h_T, are given by the solutions of the following first order conditions (FOCs):

$$\begin{aligned} \tau_T^c &\Rightarrow \frac{\partial V_T}{\partial \tau_T} = 0 \\ i_T^c &\Rightarrow \frac{\partial V_T}{\partial i_T} = 0 \\ e_T^c &\Rightarrow \frac{\partial V_T}{\partial e_T} = 0 \end{aligned} \tag{A.2}$$

Because of the value-added specification of the child cognitive ability production function, as defined by (4), child ability in period $T+1$ is a function of the inputs received by the child at period T. Hence, (A.2) can be rearranged, using the total differential, in the following way:

$$\begin{aligned} \tau_T^c &\Rightarrow \frac{\partial \bar{V}_T}{\partial \tau_T} + \frac{\partial V_{T+1}}{\partial ln A_{T+1}} \times \frac{\partial ln A_{T+1}}{\partial \tau_T} = 0 \\ i_T^c &\Rightarrow \frac{\partial \bar{V}_T}{\partial i_T} + \frac{\partial V_{T+1}}{\partial ln A_{T+1}} \times \frac{\partial ln A_{T+1}}{\partial i_T} = 0 \\ e_T^c &\Rightarrow \frac{\partial \bar{V}_T}{\partial e_T} + \frac{\partial V_{T+1}}{\partial ln A_{T+1}} \times \frac{\partial ln A_{T+1}}{\partial e_T} = 0 \end{aligned} \tag{A.3}$$

where $\bar{V}_T$ is the current utility in period T:

$$\bar{V}_T = \alpha_1 ln(TT - h_T - \tau_T) + \alpha_2 ln(w_T h_T + I_T - pi_T - e_T) + \alpha_3 ln(A_T)$$

The corresponding derivatives are given by the following expressions:

$$\frac{\partial \bar{V}_T}{\partial \tau_T} = \frac{-\alpha_1}{TT - h_T - \tau_T} \tag{A.4}$$

$$\frac{\partial V_{T+1}}{\partial lnA_{T+1}} \times \frac{\partial lnA_{T+1}}{\partial \tau_T} = (\beta\rho\alpha_3)\left(\frac{\delta_{1T}}{\tau_T}\right) \tag{A.5}$$

$$\frac{\partial \bar{V}_T}{\partial i_T} = \frac{-p\alpha_2}{w_T h_T + I_T - pi_T - e_T} \tag{A.6}$$

$$\frac{\partial V_{T+1}}{\partial lnA_{T+1}} \times \frac{\partial lnA_{T+1}}{\partial i_T} = (\beta\rho\alpha_3)\left(\frac{\delta_{2T}}{i_T}\right) \tag{A.7}$$

$$\frac{\partial \bar{V}_T}{\partial e_T} = \frac{-\alpha_2}{w_T h_T + I_T - pi_T - e_T} \tag{A.8}$$

$$\frac{\partial V_{T+1}}{\partial lnA_{T+1}} \times \frac{\partial lnA_{T+1}}{\partial e_T} = (\beta\rho\alpha_3)\left(\frac{\delta_{3T}}{e_T}\right) \tag{A.9}$$

and the FOCs become:

$$\tau_T^c \Rightarrow \frac{-\alpha_1}{TT - h_T - \tau_T} + (\beta\rho\alpha_3)\left(\frac{\delta_{1T}}{\tau_T}\right) = 0 \tag{A.10}$$

$$i_T^c \Rightarrow \frac{-p\alpha_2}{w_T h_T + I_T - pi_T - e_T} + (\beta\rho\alpha_3)\left(\frac{\delta_{2T}}{i_T}\right) = 0 \tag{A.11}$$

$$e_T^c \Rightarrow \frac{-\alpha_2}{w_T h_T + I_T - pi_T - e_T} + (\beta\rho\alpha_3)\left(\frac{\delta_{3T}}{e_T}\right) = 0 \tag{A.12}$$

The solutions for the three inputs at period T are given by:

$$\tau_T^c = \frac{\beta\delta_{1T}D_{T+1}}{\alpha_1 + \beta\delta_{1T}D_{T+1}}(TT - h_T) \tag{A.13}$$

$$i_T^c = \frac{\beta\delta_{2T}D_{T+1}}{p(\alpha_2 + \beta\delta_{2T}D_{T+1} + \beta\delta_{3T}D_{T+1})}(w_T h_T + I_T) \tag{A.14}$$

$$e_T^c = \frac{\beta\delta_{3T}D_{T+1}}{p(\alpha_2 + \beta\delta_{2T}D_{T+1} + \beta\delta_{3T}D_{T+1})}(w_T h_T + I_T) \tag{A.15}$$

where $D_{T+1} = \frac{\partial V_{T+1}}{\partial lnA_{T+1}} = \rho\alpha_3$.

These solutions can be substituted into the value function of the mother at period T, in order to get $V_T(\tau_T^c, i_T^c, e_T^c)$.

Consider now period $T-1$. The value function for this period is:

$$\begin{aligned} V_{T-1} = max_{\tau_{T-1}, i_{T-1}, e_{T-1}}\ & \alpha_1 ln(TT - h_{T-1} - \tau_{T-1}) + \alpha_2 ln(w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1} - e_{T-1}) + \\ & + \alpha_3 ln(A_{T-1}) + \\ & + E_{T-1}\beta\{\alpha_1 ln(TT - h_T - \tau_T^C) + \alpha_2 ln(w_T h_T + I_T - pi_T^C - e_T^c) + \alpha_3 lnA_T + \\ & + \beta\{\tilde{V_{T+1}} + \rho\alpha_3[\delta_{1T}ln\tau_T^C + \delta_{2T}lni_T^C + \delta_{3T}lne_T + \delta_{4T}lnA_T]\}\} \end{aligned} \tag{A.16}$$

Applying total differential, the solutions for all inputs in period $T-1$ are given by:

$$\tau^c_{T-1} \Rightarrow \frac{\partial \bar{V}_{T-1}}{\partial \tau_{T-1}} + \frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial \tau_{T-1}} = 0 \tag{A.17}$$

$$i^c_{T-1} \Rightarrow \frac{\partial \bar{V}_{T-1}}{\partial i_{T-1}} + \frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial i_{T-1}} = 0 \tag{A.18}$$

$$e^c_{T-1} \Rightarrow \frac{\partial \bar{V}_{T-1}}{\partial e_{T-1}} + \frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial e_{T-1}} = 0 \tag{A.19}$$

where

$$\bar{V}_{T-1} = \alpha_1 ln(TT - h_{T-1} - \tau_{T-1}) + \alpha_2 ln(w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1} - e_{T-1}) + \alpha_3 ln(A_{T-1})$$

and

$$\frac{\partial \bar{V}_{T-1}}{\partial \tau_{T-1}} = \frac{-\alpha_1}{TT - h_{T-1} - \tau_{T-1}} \tag{A.20}$$

$$\frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial \tau_{T-1}} = (\beta\rho\alpha_3)\left(\frac{\delta_{1T-1}}{\tau_{T-1}}\right) \tag{A.21}$$

$$\frac{\partial \bar{V}_{T-1}}{\partial i_{T-1}} = \frac{-p\alpha_2}{w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1} - e_{T-1}} \tag{A.22}$$

$$\frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial i_{T-1}} = (\beta\rho\alpha_3)\left(\frac{\delta_{2T-1}}{i_{T-1}}\right) \tag{A.23}$$

$$\frac{\partial \bar{V}_{T-1}}{\partial e_{T-1}} = \frac{-\alpha_2}{w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1} - e_{T-1}} \tag{A.24}$$

$$\frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial e_{T-1}} = (\beta\rho\alpha_3)\left(\frac{\delta_{3T-1}}{e_{T-1}}\right) \tag{A.25}$$

Substituting these expressions, the FOCs for period $T-1$ become:

$$\tau^c_{T-1} \Rightarrow \frac{-\alpha_1}{TT - h_{T-1} - \tau_{T-1}} + (\alpha_3 + \beta\alpha_3)\left(\frac{\delta_{1T-1}}{\tau_{T-1}}\right) = 0 \tag{A.26}$$

$$i^c_{T-1} \Rightarrow \frac{-p\alpha_2}{w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1} - e_{T-1}} + (\alpha_3 + \beta\alpha_3)\left(\frac{\delta_{2T-1}}{i_{T-1}}\right) = 0 \tag{A.27}$$

$$e^c_{T-1} \Rightarrow \frac{-\alpha_2}{w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1} - e_{T-1}} + (\alpha_3 + \beta\alpha_3)\left(\frac{\delta_{3T-1}}{e_{T-1}}\right) = 0 \tag{A.28}$$

The solutions for the choice variables in period $T-1$, conditional on h_{T-1}, are then:

$$\tau^c_{T-1} = \frac{\beta\delta_{1T-1}D_T}{\alpha_1 + \beta\delta_{1T-1}D_T}(TT - h_{T-1}) \tag{A.29}$$

$$i^c_{T-1} = \frac{\beta\delta_{2T-1}D_T}{p(\alpha_2 + \beta\delta_{2T-1}D_T + \beta\delta_{3T-1}D_T)}(w_{T-1}h_{T-1} + I_{T-1}) \tag{A.30}$$

$$e^c_{T-1} = \frac{\beta\delta_{3T-1}D_T}{p(\alpha_2 + \beta\delta_{2T-1}D_T + \beta\delta_{3T-1}D_T)}(w_{T-1}h_{T-1} + I_{T-1}) \tag{A.31}$$

where

$$D_T = \frac{\partial V_T}{\partial lnA_T} = \alpha_3 + \beta\delta_{4T}\underbrace{(\rho\alpha_3)}_{D_{T+1}}$$

The solutions for period $T-1$ can be substituted in (A.16) to get $V_{T-1}(\tau^c_{T-1}, i^c_{T-1}, e^c_{T-1})$. This expression can be used to write down the value function at period $(T-2)$. Using the same process described for periods T and $(T-1)$ and computing the corresponding derivatives yield the solutions for period $T-2$. The solutions for all the periods up to period $t = 1$ can be retrieved similarly.

At the end, three sequences of optimal choices can be obtained. The sequence of optimal choices for time with the child, conditional on the mother's labor supply, is given by:

$$\tau^c_T = \frac{\beta\delta_{1T}D_{T+1}}{(\alpha_1 + \beta\delta_{1T}D_{T+1})}(TT - h_T) \tag{A.32}$$

$$\tau^c_{T-1} = \frac{\beta\delta_{1T-1}D_T}{(\alpha_1 + \beta\delta_{1T-1}D_T)}(TT - h_{T-1}) \tag{A.33}$$

$$\tau^c_{T-2} = \frac{\beta\delta_{1T-2}D_{T-1}}{(\alpha_1 + \beta\delta_{1T-2}D_{T-1})}(TT - h_{T-2}) \tag{A.34}$$

$$\vdots$$

$$\tau^c_t = \frac{\beta\delta_{1t}D_{t+1}}{(\alpha_1 + \beta\delta_{1t}D_{t+1})}(TT - h_t) \tag{A.35}$$

$$\vdots$$

$$\tau^c_2 = \frac{\beta\delta_{12}D_3}{(\alpha_1 + \beta\delta_{12}D_3)}(TT - h_2) \tag{A.36}$$

$$\tau^c_1 = \frac{\beta\delta_{11}D_2}{(\alpha_1 + \beta\delta_{11}D_2)}(TT - h_1) \tag{A.37}$$

Equation (A.35) is equal to equation (7) in the text.

The sequence of the optimal non-parental child care choices, conditional on the mother's labor supply, is given by:

$$i_T^c = \frac{\beta\delta_{2T}D_{T+1}}{p(\alpha_2 + \beta\delta_{2T}D_{T+1} + \beta\delta_{3T}D_{T+1})}(w_T h_T + I_T) \tag{A.38}$$

$$i_{T-1}^c = \frac{\beta\delta_{2T-1}D_T}{p(\alpha_2 + \beta\delta_{2T-1}D_T + \beta\delta_{3T-1}D_T)}(w_{T-1}h_{T-1} + I_{T-1}) \tag{A.39}$$

$$i_{T-2}^c = \frac{\beta\delta_{2T-2}D_{T-1}}{p(\alpha_2 + \beta\delta_{2T-2}D_{T-1} + \beta\delta_{3T-2}D_{T-1})}(w_{T-2}h_{T-2} + I_{T-2}) \tag{A.40}$$

$$\vdots$$

$$i_t^c = \frac{\beta\delta_{2t}D_{t+1}}{p(\alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})}(w_t h_t + I_t) \tag{A.41}$$

$$\vdots$$

$$i_2^c = \frac{\beta\delta_{22}D_3}{p(\alpha_2 + \beta\delta_{22}D_3 + \beta\delta_{32}D_3)}(w_2 h_2 + I_2) \tag{A.42}$$

$$i_1^c = \frac{\beta\delta_{21}D_2}{p(\alpha_2 + \beta\delta_{21}D_2 + \beta\delta_{31}D_2)}(w_1 h_1 + I_1) \tag{A.43}$$

Equation (A.41) is equal to (8) in the main text.

Finally, the sequence of optimal expenditure choices is given by:

$$e_T^c = \frac{\beta\delta_{3T}D_{T+1}}{(\alpha_2 + \beta\delta_{2T}D_{T+1} + \beta\delta_{3T}D_{T+1})}(w_T h_T + I_T) \tag{A.44}$$

$$e_{T-1}^c = \frac{\beta\delta_{3T-1}D_T}{(\alpha_2 + \beta\delta_{2T-1}D_T + \beta\delta_{3T-1}D_T)}(w_{T-1}h_{T-1} + I_{T-1}) \tag{A.45}$$

$$e_{T-2}^c = \frac{\beta\delta_{3T-2}D_{T-1}}{(\alpha_2 + \beta\delta_{2T-2}D_{T-1} + \beta\delta_{3T-2}D_{T-1})}(w_{T-2}h_{T-2} + I_{T-2}) \tag{A.46}$$

$$\vdots$$

$$e_t^c = \frac{\beta\delta_{3t}D_{t+1}}{(\alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})}(w_t h_t + I_t) \tag{A.47}$$

$$\vdots$$

$$e_2^c = \frac{\beta\delta_{32}D_3}{(\alpha_2 + \beta\delta_{22}D_3 + \beta\delta_{32}D_3)}(w_2 h_2 + I_2) \tag{A.48}$$

$$e_1^c = \frac{\beta\delta_{31}D_2}{(\alpha_2 + \beta\delta_{21}D_2 + \beta\delta_{31}D_2)}(w_1 h_1 + I_1) \tag{A.49}$$

where Equation (A.47) is equal to Equation (9) and the sequence of values for D_{t+1} is reported in (10) in the main text.

Having found the solutions for the time allocation, non-parental child care and expenditure decisions, the solution for the labor supply can be computed using the same backward procedure. Equation (11) represents the optimal labor supply in each period as a function of τ_t, i_t and e_t; substituting (7),(8) and (9), it yields the optimal labor supply choice for each period t, as defined by (12).

Appendix B. The PSID data and the CDS-TD supplements

The dataset is composed by different supplements of the Panel Study of Income Dynamics (PSID) gathered in the period 1985-2007. Table B.1 summarizes the main information on availability and sources of data.

Table B.1
Availability and sources of data.

Set of Variables	**Source**	**Survey Years**	**Additional Info**
Non-parental child care	CDS	1997-2002	Retrospective questions on all arrangements used since birth and questions on arrangements used at the time of the survey
Child cognitive outcomes	CDS	1997-2002-2007	Only for children older than 3
Child demographic characteristics	CDS	1997-2002	Time-invariant (except *age*)
Maternal time with the child	CDS-TD	1997-2002	Available only for the year of the survey
Parents' hours of work	PSID	1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1999, 2001, 2003, 2005, 2007	Referred to the year before the survey
Parents' wages	PSID	1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1999, 2001, 2003, 2005, 2007	Referred to the year before the survey
Parents' non labor income	PSID	1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1999, 2001, 2003, 2005, 2007	Referred to the year before the survey
Parents' demographic characteristics	PSID	1997	Time-invariant (except *age*)

To merge PSID and CDS data I exploited information on the relationship of each CDS child with respect to the head of the household and the primary caregiver. The final sample is made up of all children aged 0-12 in 1997 without siblings and with both parents living in the household, without missing information on child's and parents' characteristics and with at least one test score measure. As summarized in Table B.2, the birth cohorts of children in this sample range from 1984 to 1996, while the terminal period of the model ($T = 13$) corresponds to 1997 for those born in 1984 and to 2009 for those born in 1996.

Table B.3 summarizes the available data for a child born in 1996. This table stresses the existence of a long time-gap of missing data, because of the structure of the surveys and the timing of the interviews. In fact, while the child care information is available for all periods, data on maternal time and child's cognitive outcomes are available only in the years of the CDS supplement, i.e., 1997, 2002 and 2007.

Table B.4 shows the average characteristics of the sample used for the estimation ($N = 417$) and the total sample of children in CDS, for whom it has been possible to derive information on their parents (3243 observations). This comparison sample includes both families with only one child and families with more children. Mothers in the sample used for the analysis spend less time with their child, work more and use a slightly higher amount of non-parental child care; moreover, they are older and more educated than the mothers in the PSID-CDS data. However, they do not differ in terms of wage at childbirth and race.

Table B.2
Cohorts of children in the final sample.

Year of Birth	Child's Age					
$t = 0$	$t = 1$	$t = 2$	$t = 3$	$\cdots$	$t = 12 = T - 1$	$t = 13 = T$
1984	1985	1986	1987	$\cdots$	1996	1997
1985	1986	1987	1988	$\cdots$	1997	1998
1986	1987	1988	1989	$\cdots$	1998	1999
1987	1988	1989	1990	$\cdots$	1999	2000
1988	1989	1990	1991	$\cdots$	2000	2001
1989	1990	1991	1992	$\cdots$	2001	2002
1990	1991	1992	1993	$\cdots$	2002	2003
1991	1992	1993	1994	$\cdots$	2003	2004
1992	1993	1994	1995	$\cdots$	2004	2005
1993	1994	1995	1996	$\cdots$	2005	2006
1994	1995	1996	1997	$\cdots$	2006	2007
1995	1996	1997	1998	$\cdots$	2007	2008
1996	1997	1998	1999	$\cdots$	2008	2009

Table B.3
Available data for a child born in 1996.

	Child's age (t)													Source	Survey Year
	1	2	3	4	5	6	7	8	9	10	11	12	13		
Non-parental child care	X	X	X	X	X	X	X	X	X	X	X	X	X	CDS	1997, 2002
Child cognitive outcomes						X					X			CDS	2002, 2007
Child demographic charact.	X					X					X			CDS	1997, 2002, 2007
Maternal time with the child	X					X								TD	1997, 2002
Parents' hours of work		X		X		X		X		X				PSID	1999, 2001, 2003, 2005, 2007
Parents' wages		X		X		X		X		X				PSID	1999, 2001, 2003, 2005, 2007
Parents' non labor income		X		X		X		X		X				PSID	1999, 2001, 2003, 2005, 2007
Parents' demographic charact.	X		X		X		X		X					PSID	1997, 1999, 2001, 2003, 2005, 2007

Appendix C. Estimation

The estimation has been done in two-stages: the parameters of the income process have been estimated in the first stage, while all remaining parameters have been estimated in the second stage. After having computed the statistics defined in Table 2 for the actual

Table B.4
Mean characteristics of the sample with respect to PSID-CDS data.

	PSID-CDS	Sample	T-test
Mother's hours of work	23.60	27.30	-11.09^{***}
	(0.14)	(0.30)	
Non-parental child care	12.21	14.80	-9.59^{***}
	(0.10)	(0.25)	
Maternal time with the child	25.83	21.16	6.18^{***}
	(0.32)	(0.68)	
Mother's wage before childbirth[a b]	10.99	11.30	−1.47
	(0.09)	(0.19)	
Mother's education	12.98	13.27	-7.87^{***}
	(0.02)	(0.03)	
Mother's age at child's birth	26.98	28.20	-15.89^{***}
	(0.04)	(0.07)	
Mother's race: white	0.61	0.61	−0.09
	(0.00)	(0.01)	
Child's gender: male	0.51	0.51	0.28
	(0.00)	(0.01)	
Child's birth weight	116.89	119.48	-8.20^{***}
	(0.11)	(0.29)	
Father's education	12.66	13.30	-17.60^{***}
	(0.01)	(0.03)	
Father's hours of work	38.66	45.27	-30.18^{***}
	(0.11)	(0.19)	
Household non labor income[a]	16.86	12.30	2.36^{**}
	(1.39)	(0.86)	
N	3243	417	

[a] Monetary variables deflated into 1997 US$.
[b] Mother's wage before childbirth refers to the year before the child was born.
*** Difference statistically significant at the $p < 0.01$ level.
** Difference statistically significant at the $p < 0.05$ level.

data, I proceed with the first-stage estimation of the income parameters. This involves the simulation of the income process, after having drawn from a standard normal distribution $N * R$ times, for every period. The statistics used to estimate these parameters are the average and standard deviation of income for all the periods, as well as the OLS coefficients of a regression where log household income is the dependent variable and the regressors are the father's education, age and race. I compute these points for both the actual and the simulated income processes. The Method of Simulated Moments estimator for this first stage minimizes an objective function where each moment condition is the distance between the income data moments and their simulated counterparts. Each moment condition is weighted using the inverse of the corresponding statistics in the data.

The second-stage involves the estimation of all remaining parameters using the same estimator. First of all, I simulate the data according to the DGP implied by the model, taking $N*R*T$ draws for wage, error in test score measure and income and $N*R$ draws for the child's and the mother's skills, as well as for the mother's preferences. Following Keane and Moffitt (1998), I re-draw the errors to simulate the income distribution using the parameters estimated in the first stage. In each period, the values for the mother's labor supply, non-parental child care and maternal time are derived using the optimal solutions implied by the model.[30] Then, after having simulated the data for all the periods, I compute the statistics defined in Table 2 from the simulated data.

The estimator used in this second-stage minimizes an objective function where each moment condition is the distance between the data statistics and the simulated counterparts, as summarized by Table 2:

$$\hat{\theta} = arg\ min\ \hat{g}(\theta)' W \hat{g}(\theta) \tag{C.1}$$

where

$$\hat{g}(\theta) = \hat{m} - \hat{M}(\theta)$$

$\hat{m}$ is the vector of statistics defined from the actual data, while $\hat{M}(\theta)$ is the vector of simulated statistics according to the model that are functions of the structural parameters to be estimated. W is a positive definite diagonal weighting matrix. The most efficient minimum distance estimator uses a weighting matrix whose elements are estimates of the inverse of the covariance matrix of the vector $\hat{m}$; this is the so-called optimal minimum distance (OMD) estimator (Cameron and Trivedi 2005, pag. 203). Since Altonji and Segal (1996) provide evidence of small sample biases in the OMD estimator, I use the diagonally weighted minimum distance estimator proposed by Blundell, Pistaferri, and Preston (2008). Given S number of moments, the weighting matrix is then defined as:

$$W = \begin{pmatrix} \hat{V}[\hat{m}_1]^{-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{V}[\hat{m}_S]^{-1} \end{pmatrix}$$

where $\hat{V}[\hat{m}]$ is estimated with non-parametric bootstrap and according to the formula (Davidson and MacKinnon 2003, p. 208):

$$\hat{V}[\hat{m}] = \left[\frac{1}{B}\right] \sum_{b=1}^{B} (\hat{m}_b^* - \bar{m}^*)(\hat{m}_b^* - \bar{m}^*)' \tag{C.2}$$

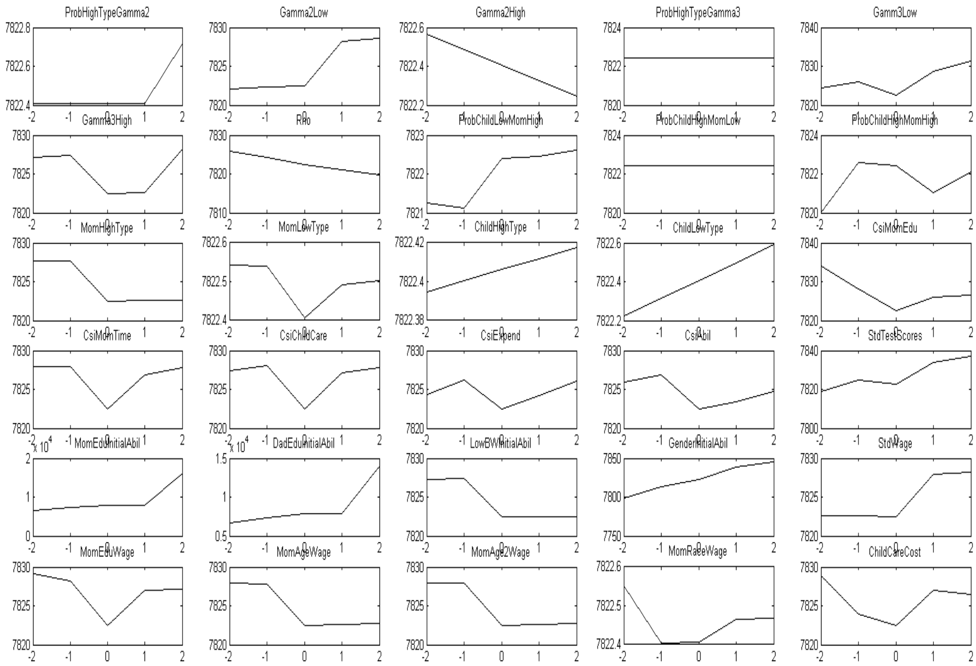
Non-parametric bootstrap (with replacement) has been implemented following Wooldridge (2002, p. 379): I used a random number generator to obtain N integers, where $N = 417$

[30] To test numerically the accuracy of the solutions given by the theoretical model, I also perform a grid search, assuming that the mother's decision to work was actually discrete. In other words, I compute the value of the demands for child care and time with the child, as well as the mother's inter temporal utility, for different levels of the mother's labor supply (with the number of hours of work ranging from 0 up to the total time endowment) and I define as optimal choices those that provide the highest utility. The solutions do not differ from the ones provided by the theoretical model.

represents the sample size of the actual data, and these integers index the observations drawn from the actual distribution of data. Repeating this process B times,[31] it yields B bootstrap samples on which the statistics defined in Table 2 can be computed: $\hat{m}_b^*$ represents a statistic computed for the sample b, while $\bar{m}^*$ is the average of the statistics across the B samples.

Figure C.1 shows the variation in the objective function (Equation (C.1)) induced by the perturbation of each estimated parameter in the vector $\hat{\theta}$.

FIGURE C.1
Objective function at the estimated parameters.



NOTES. This graph reports the values of the objective function perturbing each parameter by 2 standard deviations up and down with respect to the estimated value.

C.1. ***Standard errors.*** Non-parametric bootstrap with replacement has also been used to compute the standard errors. After having drawn B_{se} samples from the actual data,[32] I repeat the estimation of the parameters for each sample. This yields an empirical distribution of the parameters estimates, from which I can recover a bootstrap estimate of the variance, using the formula (Train 2009, pag. 201):

$$\hat{V}\left[\hat{\theta}\right] = \left[\frac{1}{B}\right] \sum_{b=1}^{B} \left(\hat{\theta}_b^* - \bar{\theta}^*\right) \left(\hat{\theta}_b^* - \bar{\theta}^*\right)' \quad \text{(C.3)}$$

[31] $B = 200$.
[32] $B_{se} = 50$

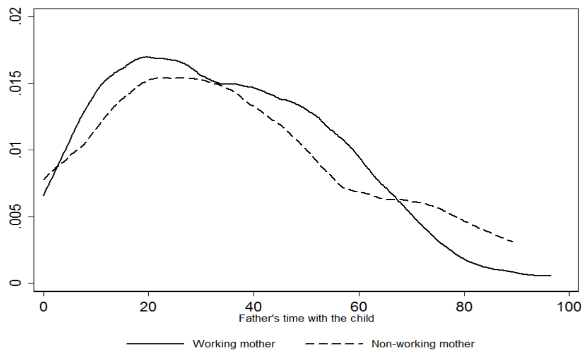
Taking the square root of (C.3) yields the bootstrap estimate of the standard errors $se_{\hat{\theta}}$. Bootstrap has been used also to compute the standard errors of transformed parameters: type proportions, preference and productivity parameters.

Appendix D. Sensitivity analysis

I test the sensitivity of the results presented in Section 6 under two main dimensions. For the sake of brevity, I report only the results concerning the parameters for maternal time and non-parental child-care time in the CAPF.

D.1. ***Mother's and father's time investments.*** The variable weekly time with the mother has been defined considering the time spells when the child was with the mother, either being the mother directly involved in child's activities or being just around and not participating. This implies that only the mother's time is productive for the child cognitive development, modeling the father's contribution to the child development process only through the father's labor income and assuming that the father's labor supply is exogenous. However, the father's time with the child can somehow respond to the mother's decision of entering the labor force. Figure D.1 shows the kernel density distribution of a father's time with the child by a mother's employment status, and suggests that it does not vary systematically with the mother's decision to work. Moreover, I check the sensitivity of the results to the definition of maternal time and I re-estimate the model including the time spells in which the child is with the mother *and* the father. Results are reported in Figure D.2 and are qualitatively similar to the ones presented in the main analysis.

Figure D.1
Father's time with the child by mother's employment status.

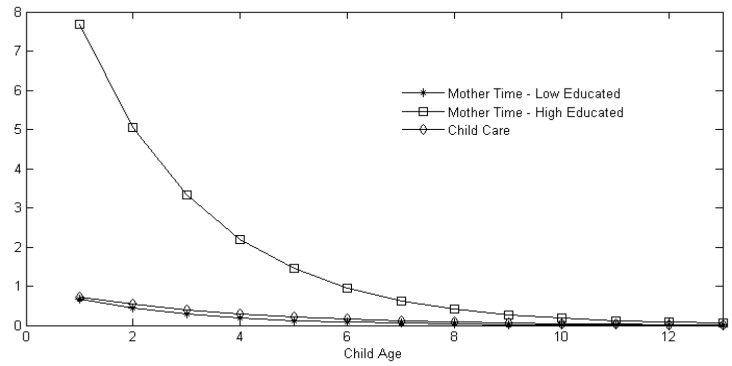


NOTE. This graph represents the Kernel-density distribution of father's time with the child by mother's employment status.

D.2. ***Child care and schooling.*** The second issue relates, instead, to the definition of non-parental child care and the impossibility to observe schooling inputs after the child enters kindergarten or primary school. In the baseline specification, since I cannot observe schooling investments, I am ruling out these inputs from the child development process. I check the sensitivity of the results in two ways. The first one consists of defining the variable for non-parental child care adding five more hours of schooling after the child turns six, in such a way that the alternative forms of care used by the mother include not

Figure D.2

Elasticity of child's ability with respect to mother's time with the child and non-parental child care if maternal time includes also time with the father.



NOTE. This graph represents the productivity parameters for maternal time (τ_t) and non-parental child care (i_t) as a function of child's age $t = 1, 2, \ldots 13$. τ includes all time spells when the child is with the mother and the ones when the child is with both the mother and the father.

only non-parental child care centers or baby sitters, but also schooling. The second test consists of modeling schooling as a total factor productivity in the CAPF and varying over time. The CAPF in this case becomes:

$$lnA_{t+1} = ln(\delta_{0t}) + \delta_{1t}ln\tau_t + \delta_{2t}lni_t + \delta_{3t}lne_t + \delta_{4t}lnA_t \tag{D.1}$$

where

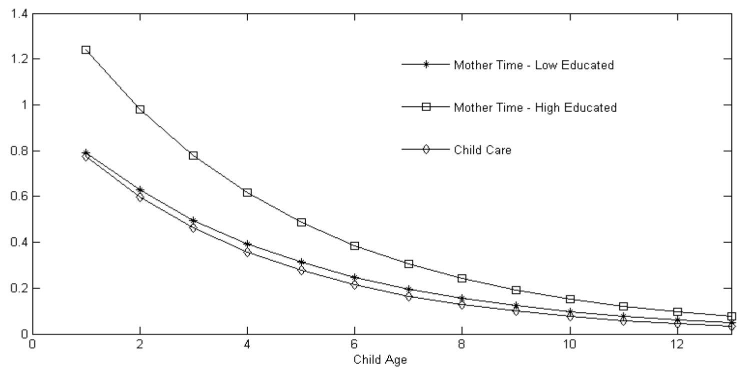
$$\delta_{0t} = \begin{cases} 1 & \text{if} \quad t <= 5 \\ exp(\xi_0) & \text{if} \quad t >= 6 \end{cases}$$

and ξ_0 is an additional parameter to be estimated.

Results for the productivity parameters of non-parental child care and maternal time with the child in the case in which the variable non-parental child care also includes schooling time are reported in Figure D.3. Figure D.4, instead, shows the results of the estimation of the model in which schooling is specified as a total factor productivity. In both cases, they are qualitatively similar to those shown in Figure 8.

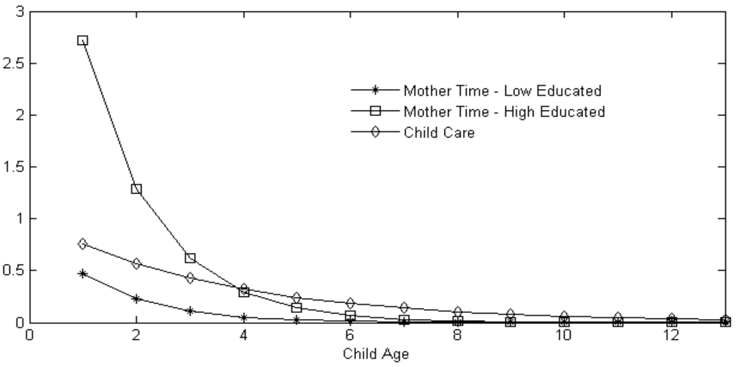
Figure D.3

Elasticity of child's ability with respect to mother's time with the child and non-parental child care if non-parental child care includes also schooling time.



NOTE. This graph represents the productivity parameters for maternal time (τ_t) and non-parental child care (i_t) as a function of child's age $t = 1, 2, \ldots 13$. i_t, after age 5, is defined including also 5 additional hours of schooling.

Figure D.4
Elasticity of child's ability with respect to mother's time with the child and non-parental child care with schooling as TFP.



NOTE. This graph represents the productivity parameters for maternal time (τ_t) and non-parental child care (i_t) as a function of child's age $t = 1, 2, \ldots 13$. The specification of the model allows an additional parameters in the CAPF representing the total factor productivity of school after age 6.